DISCUSSION OF TIDES IN BOSTON HARBOR

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APPENDIX No. 5.

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PREPARED

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UNITED STATES COAST SURVEY.

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FROM THE COAST SURVEY REPORT FOR 1868.

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CAMBRIDGE, MASSACHUSETTS, *May* 2, 1870.

DEAR SIR: I have the honor to submit the following report of a discussion of the Boston dry-dock tide observations. On account of the completeness of the set of observations and the importance of the tide-station, as representing a peculiar, and, in relation to the tidal theory, a very interesting, type of tides, extending along the whole coast of New England, it has been thought advisable to give all the labor to the discussion necessary to obtain the most accurate results, and also to give a somewhat full report of them. Some applications of the results have also been made to theory, and practical formulæ and tables constructed for the prediction of the tides.

THE OBSERVATIONS AND LOCALITY.

1. The series of observations used in this discussion embrace a period of nineteen years, commencing with the 1st of July, 1847, and extending to the 1st of July, 1866. The series is very nearly complete, the observations of the times and heights of both high and low water having been made with great regularity both day and night, and for every day of the week; so that it rarely happened that even a single observation was lost. Only on two occasions were there any interruptions of any consequence in the whole series. During the latter part of May, and the first part of June, 1854, one week's observations were lost, owing to the observer's being on duty to check the mob at the time of the Burns trial. The observations of ten days, also, of the latter part of January, 1865, were lost on account of the illness of the observer.

The first part of the series of observations was made with a tide-gauge consisting of marks designated by copper figures set in the stone wall at the entrance to the dry-dock. The observer commenced usually about a half hour before each high and low water and noted the readings every five minutes until the stand, then the duration of the stand, and then again he noted the readings every five minutes until he felt sure the tide had changed. In 1860 a box-gauge was substituted, which was used until the series was closed, on the 30th of September, 1866, the full series having been commenced in June and continued about nineteen years and four months.

The observations seem to have been accurately made, so far as can be judged from the grouping of them in the discussion; but this is rather an uncertain criterion, since numerous astronomical inequalities, and also meteorological irregularities, necessarily cause a considerable range in the observations of each group. The most accurate test of the faithfulness of the observer is found in the slight irregularities caused by the diurnal tide. These irregularities in both the heights and times of the tide at the Boston dry-dock are quite small, and only sensible to ordinary inspection of the observations at certain periods; yet these irregularities can generally be readily distinguished in the recorded observations, amid all the other numerous irregularities, at such times as theory indicates that they should be most easily seen. It is not probable that the observer knew either the periods of these irregularities, or their character, and hence their occurring in the record must have resulted from a careful observation of the tides.

This discussion has not been made from the original records of the observer, but from what are called the first reductions of the tide observations, in which are given the apparent times of the moon's meridian transit, and the apparent times and absolute heights of high and low water, and the lunitidal intervals, as obtained from the observer's record. These reductions, so far as I had any means of testing them, seem to have been carefully and accurately made.

2. On account of the shallowness of the greater part of the harbor, and even of the bay for many miles beyond, and the interruptions of numerous islands, and the narrowness of the channel leading to the Boston dry-dock, the tide-station must be regarded as a somewhat inland one, and some of the characteristics of river-stations should be found in the observed tide. It is seen, from consulting the Coast Survey chart of the bay and harbor, that the average depth of the channel in

the harbor for ten miles ranges mostly from five to ten fathoms, and beyond that in the open bay for many miles there is only a very gradual increase of the depth of the sea. The vast expanse, also, of comparatively shallow ocean over the Banks of Newfoundland, over which the greater part of the tide originating in the deeper ocean has to travel, no doubt affects the character of the tides. In any casual considerations of the character of the tides and of the results of this discussion, these general circumstances of the tide-station should be considered, but in any critical study of them, of course, the charts of the Coast Survey should be consulted, in which all the minute circumstances are accurately laid down.

EXPRESSIONS OF THE DISTURBING FORCES.

3. In the discussion of tide observations it is necessary to have some form of expression of the disturbing forces, and to know something of the tidal expressions corresponding with them. Theory must furnish the arguments to be used in any discussion, but different forms of expression give different arguments. In the complete solution of the tidal problem it is necessary to have a development of the disturbing forces into a series of terms containing angles, which increase in proportion to the time, and the corresponding tidal expression has then a similar form; but very accurate approximate tidal expressions may be obtained in which the expressions contain circular arguments, but in which the angles do not increase exactly in proportion to the time. Such expressions may be obtained which contain a much smaller number of sensible terms than in the case in which the development is required to contain only terms with angles increasing exactly in proportion to the time; and the arguments in the expression, still being circular arguments, are much preferable to parallax and declination arguments, since all the observations, within certain equal limits of the argument, have nearly the same number of observations; and the results obtained from a discussion of the observations in this way are of much more importance in any theoretical study and investigation of the tides, since the constant coefficients and the angles of epoch show the relations between each term in the tidal expression and the corresponding term in the disturbing force.

4. Since the forms of expression of the disturbing forces used in this discussion, and likewise the notation, are for the most part entirely different from those contained in any treatise on the tides to which reference could be made, it is necessary to give them here.

If we put

Ω = the potential of the moon's disturbing force;
μ = its mass;
r = its distance from the center of the earth;
ψ = its right ascension;
v = its declination;
ϖ = the terrestrial longitude of any place;
θ = its polar distance;
$n\,t$ = the earth's rotatory motion;

we have, from the development of Ω,

$$\Omega = \Sigma_s \mathrm{N}_s \cos s\,(n\,t + \varpi - \psi) \tag{1}$$

in which

$$\left\{\begin{aligned} \mathrm{N}_0 &= \frac{\mu}{4\,r^3}(1 - 3\cos^2\theta)(1 - 3\sin^2\mathrm{v}) \\ \mathrm{N}_1 &= \frac{3\,\mu}{4\,r^3}\sin 2\,\theta \sin 2\,\mathrm{v} \\ \mathrm{N}_2 &= \frac{3\,\mu}{4\,r^3}\sin^2\theta\cos^2\mathrm{v} \\ \mathrm{N}_3 &= \frac{3\,\mu}{8\,r^3}\sin^3\theta\cos^3\mathrm{v} \end{aligned}\right. \tag{2}$$

There is also a small term depending upon the fourth power of the moon's distance, which produces a sensible effect, of the form

$$\mathrm{N} = \frac{15\,\mu}{4\,r^4}\cos\theta\sin^2\theta\cos^2\mathrm{v}\sin\mathrm{v} \tag{3}$$

5. If we likewise put

$$\Omega' = \text{the potential of the sun's disturbing force,}$$

and let μ', r', ψ', and v' denote the same with regard to the sun, which the same letters without an accent do with regard to the moon, we shall have

$$\Omega' = \Sigma_s N'_s \cos s\,(n\,t + \varpi - \psi') \tag{4}$$

in which the expressions of N'_s are the same as those of N_s with μ, r, ψ, and v accented.

In the preceding expressions the origin of t must be such as to make the angle $n\,t + \varpi - \psi$ or $n\,t + \varpi - \psi'$ equal to $i\pi$, that is, some multiple of π, when the moon or sun is on the meridian of the place with regard to which the force is considered. The values of s may be 0, 1, 2, &c., but there are no terms producing sensible effects in which s is greater than 3.

6. In the comparison of tides with the forces producing them, it is necessary to either analyze the result and tide of the moon and sun in each port into its component parts, or to have the resultant of the component forces of the moon and sun with which to compare them. The latter is preferable in tidal discussions and investigations, since the developed expressions of the resultant of the forces of the moon and sun being obtained, and all the constants accurately determined, these expressions, depending mostly upon celestial circumstances, serve, with a very few convenient modifications, for every port; whereas if the former method is adopted, a troublesome analysis, and a determination of the constants belonging to each component of the tide, must be made for each port.

By combining the preceding components, we get

$$\Omega + \Omega' = \Sigma_s \sqrt{N_s^2 + N'^2_s + 2\,N_s N'_s \cos s\,(\psi - \psi')}\,\cos s\,(n\,t + \varpi - \psi + \beta_s) \tag{5}$$

in which

$$\tan s\,\beta_s = \frac{N'_s \sin s\,(\psi - \psi')}{N_s + N'_s \cos s\,(\psi - \psi')} \tag{6}$$

7. If we put

$$\Omega_s = \text{the part of the preceding expression belonging to } s,$$

its development may be expressed in the following form:

$$\Omega_s = C_s\,\Sigma_i\,P_i \cos \eta_i \cos s\,(n\,t + \varpi - \psi + \beta_s) \tag{7}$$

in which the angles η_i and $n\,t + \varpi - \psi + \beta_s$ do not increase exactly in proportion to the time on account of the variable motions of the moon and sun in their orbits, and the obliquity of the ecliptic. The latter angle also varies with the changing value of β_s which expresses the angle in right ascension between the moon and the position of a disturbing body which would represent the resultant of that part of the forces of the moon and sun belonging to the characteristic s. In the preceding expression C_s is the constant or average value of the coefficient of $\cos s\,(n\,t + \varpi - \psi + \beta_s)$, and is independent of any of the inequalities. Its value depends upon terrestrial circumstances, and consequently is different in different ports. The constants P_i and the angles η_i depend upon celestial circumstances only, and consequently are the same for every port. The constants P_i are different for different values of s, and should be denoted by $P_{(s,\,i)}$ when it is necessary to distinguish them. P_0 is the constant of integration and is equal to unity.

8. We shall now give the angles and the numerical values of the constants, and also the mean values of the first derivatives of the angles in terms of the radius, belonging to the principal terms in the preceding expression of Ω_s for each value of s. In the expressions of the angles the following notation is used:

v = the moon's mean anomaly;
v' = that of the sun;
φ = the moon's longitude;
φ' = that of the sun;
ω = the longitude of the moon's ascending node.

From the notation (§§ 4 and 5) we also have

$\psi - \psi'$ = the difference in right ascension between the moon and sun, usually expressed by the apparent time of moon's transit.

We shall also put, for the mean values of r and r',

$$(8)\qquad \begin{cases} Z=\dfrac{3\mu}{4r^3} \\ Z'=\dfrac{3\mu'}{4r'^3} \\ e=\dfrac{Z'}{Z} \end{cases}$$

If we put, in terms of the earth's mass,

$$(9)\qquad \mu=.013+\delta\mu$$

in which $\delta\mu$ is the correction of the assumed mass of the moon, we shall have

$$(10)\qquad e=.4380-33.8\,\delta\mu$$

9. With the preceding constants and notation, when $s=0$, (7) gives

$$(11)\qquad \Omega_0=C_0\,\Sigma_i\,P_i\cos\eta_i$$

in which, omitting the correction of the moon's mass,

$$(12)\qquad \begin{cases} C_0= .254\,(1-3\cos^2\theta)(1+e)\,Z & \\ P_2= .114, & \eta_2=v \\ P_3= .217, & \eta_3=2\varphi \\ P_4= .016, & \eta_4=v' \\ P_5= .095, & \eta_5=2\varphi' \\ P_6=-.025, & \eta_6=\omega \end{cases}$$

The term belonging to $i=1$, in this case, is wanting.

10. When $s=1$, the development of the resultant of the moon and sun in the general form of (7) is not sufficiently convergent for practical purposes, and therefore expressions must be obtained for the moon and sun separately in this case. The only terms, in the case of the moon, which we shall have occasion to use in this discussion, may be most conveniently expressed in the following form, β vanishing in this case:

$$(13)\qquad \Omega_1=C_1\sin\varphi\cos(nt+\varpi-\psi)$$

in which

$$(14)\qquad C_1=.731\sin 2\theta\,Z$$

In the case of the sun we shall likewise have

$$(15)\qquad \Omega'_1=C'_1\sin\varphi'\cos(nt+\varpi-\psi')$$

in which

$$(16)\qquad C'_1=.731\,e\sin 2\theta\,Z$$

11. When $s=2$, (7) gives

$$(17)\qquad \Omega_2=C_2\,\Sigma_i\,P_i\cos\eta_i\cos 2(nt+\varpi-\psi+\beta_2)$$

in which

$$(18)\qquad \begin{cases} C_2 = .9564\sqrt{1+e^2}\sin^2\theta\,Z & & \\ P_1 = .4305-24.0\,\delta\mu, & \eta_1=2(\psi-\psi'), & D_t\eta_1=.426 \\ P_2 = .1521+\ \ 3.6\,\delta\mu, & \eta_2=v, & D_t\eta_2=.229 \\ P_3 = .0985+\ \ 1.0\,\delta\mu, & \eta_3=2\varphi, & D_t\eta_3=.460 \\ P_4 = .0093-\ \ 1.0\,\delta\mu, & \eta_4=v', & \\ P_5 = .0053-\ \ 1.0\,\delta\mu, & \eta_5=2\varphi', & \\ P_6 =-.0375, & \eta_6=\omega, & \\ P_7 = .0375, & \eta_7=2\varphi-\omega, & D_t\eta_7=.462 \\ P_8 = .0085, & \eta_8=\eta_1+\eta_2, & D_t\eta_8=.655 \\ P_9 = .0085, & \eta_9=\eta_1-\eta_2, & D_t\eta_9=.197 \\ P_{10}=-.0470+\ \ 4.7\,\delta\mu, & \eta_{10}=2\eta_1, & D_t\eta_{10}=.852 \end{cases}$$

The unit of time in the preceding derivatives is one solar day.

12. When $s=3$, (7) gives only one term producing any sensible effect upon the tides, which may be expressed by

$$(19)\qquad \Omega_3=C_3\cos 3(nt+\varpi-\psi)$$

in which

$$(20)\qquad C_3=.0146\sin^3\theta\,Z$$

13. Putting

(21) $$i = 2\, D_t\, (n\, t - \psi + \beta_2)$$

it is necessary, in the various tidal expressions, to know the principal constants in the following expressions:

(22) $$i = \Sigma_i\, U_i \cos \eta_i, \text{ and } \frac{2\, \beta_2}{i} = \Sigma_i\, Q_i \sin \eta_i$$

The principal of these are

(23) $$\begin{cases} U_0 = 12.142, & \\ U_1 = \quad .1742 - 13.2\, \delta\, \mu, & Q_1 = 52^m.5 - .4034\, \delta\, \mu \\ U_2 = -.0500, & \\ U_3 = \quad .0477 - \ 0.5\, \delta\, \mu, & Q_3 = \ 2^m.2 - 148\, \delta\, \mu \end{cases}$$

The other values of U_i and Q_i are small, and, their effects being generally insensible, they are omitted here.

TIDAL EXPRESSIONS.

14. If we put

$Y =$ the height of the tide at any time above mean level;

$L =$ the lunitidal interval in solar time;

$\rho = n\, t + \varpi - \psi + \beta$;

$\tau_i =$ the time the maximum of any inequalities in the tides follows the maximum of the corresponding inequality of the disturbing force,

theory gives the following tidal expressions corresponding with the general expression (7) of the potential of the disturbing force:

(24) $$Y_s = K_s\, \Sigma_i\, R_i \cos(\eta_i - \alpha_i) \cos(s\, \rho - l) = A_s \cos(s\, \rho - l)$$

in which

(25) $$\begin{cases} (1 + F)\, R_i = P_i - E\, U_i \\ \qquad \alpha_i = (\tau - B_0)\, D_t\, \eta_i \end{cases}$$

and

(26) $$L_s = \Sigma_i\, B_i \sin(\eta_i - \varepsilon_i)$$

in which

(27) $$\begin{cases} B_i = \sqrt{M_i^2 + N_i^2} \\ \tan(\varepsilon_i - \alpha_i) = -\dfrac{N_i}{M_i} \\ M_i = -Q_i + \dfrac{1.035}{s}\, E\, P_i\, D_t\, \eta_i - \dfrac{0.164}{s}\, R_i\, D_t\, \eta_i \\ N_i = F'\, R_i \end{cases}$$

The value of B_0 is the *mean establishment of the port* belonging to the assumed transit.

15. Of the constants in these expressions, $E = D\, i\, K$ expresses the ratio between any change in i, the velocity with which the phase of the tide changes, and the corresponding change in the coefficient of the tide, and consequently the terms depending upon E show the effect of any change in the period of oscillation from the mean period. The constant F depends upon that part of friction in the theory which is supposed to affect the tides in a greater ratio than the first power of the velocity, and consequently affects the large tides more in proportion than the small ones, and, neglecting terms of a third order in the developments, affects the inequalities of the tides in a constant ratio. The terms depending upon F' express the corresponding effect upon the lunitidal intervals. All the constants in the preceding expressions have different values for different values of s, and they should be written $R_{(s,\, i)}$, $\alpha_{(s,\, i)}$, &c., when it is necessary to distinguish them.

16. From the second of (25) we get

(28) $$\tau_i = B_0 + \frac{\alpha_i}{D_t\, \eta_i}$$

The value of τ given by this expression has been called the *age of the tide from the heights.*

If we likewise put

(28^1) $$\tau'_i = B_0 + \frac{\varepsilon_i}{D_t\, \eta_i}$$

the value of τ'_1 in this expression has been called the *age of the tide from the times.*

The values of τ and τ' cannot be the same unless $a_i=\varepsilon_i$, which can only be the case when F' in (27) is equal 0. Hence, the difference depends on friction.

17. In the equilibrium theory all terms depending upon $U_{(i)}\,D_t\,\eta$ and upon friction vanish, and (24) gives

(29) $$Y_s=K_s\,\Sigma_i\,P_i\cos\eta_i$$

in which, in the case of an ocean covering the whole earth with a fluid of insensible density, putting g for gravity,

$$K_s=\frac{C_s}{g}$$

In the case of nature the true values of K_s differ a little from these, but these may be regarded as very near approximate values. With the values of C_s, (12), (14), (18), (20), and with the value of

(30) $$\frac{Z}{g}=(.9224+71.1\,\delta\mu)\text{ ft.},$$

we get for the port of Boston, where $\theta=47^\circ\,40'$, by neglecting the correction of the moon's mass,

(31) $$K_0=-0.117\text{ ft.},\ K_1=0.711\text{ ft.},\ K_2=0.528\text{ ft.},\ K_3=0.006\text{ ft.}$$

This value of K_0 is the mean or constant amount by which the mean level of the ocean is elevated by the moon and sun above the level which the water would assume in the case of no disturbing force. This value of K_0 may be also used in the hydrodynamic theory, since, when $s=0$, the oscillations depend upon the angles η_i in the expressions of Y_0, that is, upon the parallax and declination of the moon and sun, and hence are oscillations of long period compared with the diurnal and semidiurnal oscillations. They are called by Laplace *oscillations of the first kind.*

18. When $s=1$, (24) gives as the tidal expressions corresponding to (13) and (15)

(32) $$\begin{cases} Y_1=K_1\sin(\varphi-a)\cos(\rho-l_1) \\ Y'_1=K'_1\sin(\varphi'-a')\cos(\rho'-l'_1) \end{cases}$$

in which

$$\rho'=nt+\varpi-\psi'$$

In this case we do not know the relation between C_1 and K_1, and consequently K_1 can only be determined from observation. In this case the period of the oscillations is one day, and the oscillations are called by Laplace *oscillations of the second kind.*

19. When $s=2$, (24) gives as the tidal expression corresponding with (17)

(33) $$Y_2=K_2\,\Sigma_i\,R_i\cos(\eta_i-a_i)\cos(2\rho-l_2)=A_2\cos(2\rho-l_2)$$

In this case the mean period of the oscillations is half of a mean lunar day, giving rise to the semidiurnal tides. These Laplace calls *oscillations of the third kind.* The expression of L_2 in this case is derived from (26) and (27), putting $s=2$, and using the values of P_i, $D_t\,\eta_i$, U_i, and Q_i in (18) and (23).

20. If we change the assumed transit from which L_2 is reckoned n transits forward, then the constant B_0 is diminished n times $12^h\,25^m.24$, and the whole of the corresponding change k in the expression of L_2 is

(34) $$k=-n\,(12^h\,25^m.24+0^m.4\cos\eta_1+3^m.0\cos\eta_2-2^m.3\cos\eta_3\ldots)$$

This expression is only approximate when a change of several transits is made.

21. When $s=3$, the tidal expression corresponding with (19) is

(35) $$Y_3=K_3\cos(3\rho-l_3)$$

in which K_3 must be determined from observation. In this case the period of the oscillations is one-third of a day, and, in accordance with Laplace's method of designating them, they may be called *oscillations of the fourth kind.*

There may be local circumstances, such as the shallowness of the harbor or river, producing quarter-day tides, but these do not depend upon any sensible term in the disturbing force.

The tidal expression of the small term in the moon's disturbing force depending upon the fourth power of the distance (3), since $\sin v=\sin\varepsilon\sin\varphi$, neglecting the inequality of the node, is of the form

(36) $$Y''=K''\sin(\varphi-a'')$$

22. The preceding are the tidal expressions belonging to the different kinds of oscillation taken separately; but for a comparison of observations with theory, and also for the purpose of prediction formulæ, it is necessary to have expressions of the height of the tide and of the lunitidal intervals belonging to the resultant of all the oscillations. If we put

H_0=the height of the mean level of the sea above any assumed zero plane, in the case of no disturbing force,

then the expression of the height of high water at any time is $H_0+\Sigma_s Y_s$. If we put

H_n=the absolute height of the tide at the nth high or low water, $n=1$ belonging to the high water depending upon the upper transit;

λ_n=the corresponding lunitidal interval;

and also put

$$(37)\qquad \begin{cases} \lambda_n = L_2 + q_n \\ \varDelta = L_2 - L_1 \\ \varDelta' = L_2 - L_3 \end{cases}$$

we get, by combining the oscillations,

$$(38)\qquad \begin{cases} H_n = H_0 + A_0 + A_1 \cos \varDelta \cos q_n - A_1 \sin \varDelta \sin q_n + A_2 \cos 2\, q_n \\ \qquad + A_3 \cos 3\, \varDelta' \cos 3\, q_n - A_3 \sin 3\, \varDelta' \sin 3\, q_n \end{cases}$$

in which q_n must satisfy the conditions

$$(39)\qquad \begin{cases} 0 = A_1 \cos \varDelta \sin q_n + A_1 \sin \varDelta \cos q_n + 4\, A_2 \sin q_n \cos q_n \\ \qquad + 3\, A_3 \cos 3\, \varDelta' \sin 3\, q_n + 3\, A_3 \sin 3\, \varDelta \cos 3\, q_n \end{cases}$$

In general, there are four values of q_n which satisfy these conditions, two belonging to high waters and two to low waters. When A_1, however, is very large, there are only two values which satisfy them, and then there is only one high and one low water in a day.

The value of A_3 is always small, and when A_1 is also small, as it is at all ports in the North Atlantic, the value of q at high waters is so small that we can put $\cos q=1$, and at low waters so nearly equal to $\frac{1}{2}\pi$ that we can put $\sin q=1$. The preceding conditions then give

$$(40)\qquad \begin{cases} H_1 = H_0 + A_0 + A_1 \cos \varDelta - A_1 \sin \varDelta \sin q_1 + A_2 + A_3 \cos 3\, \varDelta' - A_3 \sin 3\, \varDelta' \sin 3\, q_1 \\ H_2 = H_0 + A_0 + A_1 \cos \varDelta \cos q_2 - A_1 \sin \varDelta - A_2 + A_3 \cos 3\, \varDelta' \cos 3\, q_2 + A_3 \sin 3\, \varDelta' \\ H_3 = H_0 + A_0 - A_1 \cos \varDelta - A_1 \sin \varDelta \sin q_3 + A_2 - A_3 \cos 3\, \varDelta' - A_3 \sin 3\, \varDelta' \sin 3\, q_3 \\ H_4 = H_0 + A_0 + A_1 \cos \varDelta \cos q_4 + A_1 \sin \varDelta - A_2 + A_3 \cos 3\, \varDelta' \cos 3\, q_4 - A_3 \sin 3\, \varDelta' \end{cases}$$

They also give

$$(41)\qquad \begin{cases} \sin q = -\dfrac{A_1 \sin \varDelta + 3\, A_3 \sin 3\, \varDelta'}{4\, A_2 + A_1 \cos \varDelta + 9\, A_3 \cos 3\, \varDelta'} \text{ at high waters, and} \\[2ex] \cos q = -\dfrac{A_1 \cos \varDelta + 3\, A_3 \cos 3\, \varDelta'}{4\, A_2 + A_1 \sin \varDelta + 9\, A_3 \sin 3\, \varDelta'} \text{ at low waters.} \end{cases}$$

When all the necessary constants are determined, the preceding equations (40) and (41) give H_n and q_n, and then when L_2 is determined, (37) gives λ_n, L_1, and L_3.

All the preceding expressions are taken from the manuscript of a forthcoming paper on the theory of the tides, in which they are more fully explained, but it would be impossible to give a complete and detailed explanation of them here. Few of them, however, depend upon any peculiar theory, and most of them can be verified by any one.

THE OBJECT AND PLAN OF THE DISCUSSION.

23. The object of the following discussion is, first, to obtain directly from observation the constants of all the principal terms entering into the preceding expressions of H_n and λ_n, which, it will be seen by referring to (24) and (26), comprise all the constants $R_{(s,\,i)}$, $\alpha_{(s,\,i)}$, $B_{(s,\,i)}$, and $\varepsilon_{(s,\,i)}$, belonging to each one of the angles in the expressions; and, secondly, to obtain from these the general constants E, G, F, and F′ in the expressions (25) and (27), expressing the relations between all the preceding constants, so that they may all be made to depend upon these few constants. It is not proposed to determine merely so many of the former as are necessary to determine the latter, thus

making all the rest depend upon theory, but to determine all that are of much importance practically, so that they may be used in testing the accuracy of the general theoretical expressions, and for constructing approximate formulæ of prediction independently of any theoretical relations between the constants, or used by any investigator for verifying and improving any tidal theory. All the constants and relations being determined which are of theoretical importance, the most convenient practical formulæ will be constructed from the results for the prediction of the times and heights of high and low water, together with tables for facilitating the computations.

The plan adopted for determining the constants from the observations is to apply Lubbock's method of averages to circular arguments throughout, instead of to arguments of parallax and declination, and then to use the constants thus obtained to determine the constants belonging to any other forms of expression into which it may be thought advisable to put the results. As the quantities H_n and λ_n are the only ones which are directly observed, corresponding to any given time or values of the arguments, these must be determined from observations for all parts of the arguments separately by so grouping the observations that the effects belonging to all the other arguments are eliminated, and then, by means of the conditions (33), (37), (40), and (41), the constants belonging to each argument, as well as the general constants independent of any arguments, can be determined.

To obtain the values of H_n and λ_n belonging to the different parts of any argument η_n alone, all the observations within certain limits of the argument as from η'_i to η''_i have been grouped together, and the averages of all these have been taken as the normal values of H_n and λ_n belonging to the averages of all the corresponding values of the argument, which, when the number of the observations is considerable, does not differ much from $\frac{1}{2}(\eta'_i+\eta''_i)$, the mean of the two limits. If these limits should be somewhat wide, a slight correction to the averages of the observations may be necessary, which is easily applied. When the observations extend over a long period, and have been regularly made, the effects of all the inequalities belonging to other arguments are completely eliminated, since the periods of all the other arguments being different, the observations falling within certain limits of any one argument, are equally distributed through all parts of the other arguments, and all the plus and minus effects cancel one another; and this is especially the case in a series of nineteen years, which is very nearly a multiple of the periods of all the principal arguments in the tidal expressions. In a long series, also, the inequalities due to the winds and barometric changes, and whatever other abnormal disturbances there may be, are very nearly eliminated.

24. If the normal values of λ_n and H_n have been obtained from all the observations without regard to the arguments of the inequalities, then all the inequalities in λ_n and H_n depending upon these arguments disappear, as also A_1 and the inequalities of A_0, A_2, and A_3 in (40) and (41). Putting

$$\lambda'_1=\tfrac{1}{2}(\lambda_1+\lambda_3) \qquad H'_1=\tfrac{1}{2}(H_1+H_3)$$
$$\lambda'_2=\tfrac{1}{2}(\lambda_2+\lambda_4) \qquad H'_2=\tfrac{1}{2}(H_2+H_4)$$

we in this case obtain from (40),

$$(42)\qquad \begin{cases} \tfrac{1}{2}(H'_1+H'_2)=H_0+K_0+K_3\cos 3\Delta'\cos 3q_2-K_3\sin 3\Delta'\sin 3q_1 \\ \tfrac{1}{2}(H'_1-H'_2)=K_2-K_3\cos 3\Delta'\cos 3q_2-K_3\cos 3\Delta'\cos 3q_2 \\ \tfrac{1}{2}(H_1-H_3)=K_3\cos 3\Delta' \\ \tfrac{1}{2}(H_2-H_4)=K_3\sin 3\Delta' \end{cases}$$

and, from (41), omitting 9 A_3 in comparison with 4 A_2 in these very small quantities,

$$(43)\qquad \begin{cases} \sin q_1=-\dfrac{3K_3\sin 3\Delta'}{4K_2} \\ \cos q_2=-\dfrac{3K_3\cos 3\Delta'}{4K_2} \end{cases}$$

Since $\sin q_1=\sin q_3$, and $\sin q_2=\sin q_4$, we have $\Sigma_n q_n=\frac{3}{2}\pi$, and hence from the first of (37) we get $L_2=\frac{1}{4}(\Sigma_n\lambda_n-\frac{3}{2}\pi)$ when λ_n are all reckoned from the upper transit; but if two of them, as is usual, are reckoned from the lower transit, putting $\pi=12^h\,25^m.24$ in solar time, we get, since $L_2=B_0$ in this case,

$$(44)\qquad B_0=\tfrac{1}{4}(\Sigma_n\lambda_n-12^h\,25^m.24)=\tfrac{1}{2}(\lambda'_1+\lambda'_2-6^h\,12^m.62)$$

which is the *mean establishment of the port.*

In the first two equations of (40), the terms depending upon A_3 being the products of two factors, which are both generally small, are, for the most part, entirely insensible, and may be omitted. If we then put

$$H'_0 = \text{the mean height of the sea above the assumed zero plane,}$$

we shall have

$$H'_0 = H_0 + K_0 = \tfrac{1}{2}(H'_1 + H'_2) \tag{45}$$

25. If we obtain the values of λ_n for any values of η_i by grouping the observations in such a manner that all the inequalities belonging to the other arguments are eliminated, we have from (26) for high waters,

$$\lambda'_1 - B_0 - q_1 = \Sigma_i\, B_i \sin(\eta_i - \varepsilon_i) = \Sigma_i\,(M_i \sin\eta_i + N_i \cos\eta_i) \tag{46}$$

and for low waters,

$$\lambda'_2 - B_0 - q_2 + k = \Sigma_i\, B_i \sin(\eta_i - \varepsilon_i) = \Sigma_i\,(M_i \sin\eta_i + N_i \cos\eta_i) \tag{47}$$

in which

$$B_i = \sqrt{M_i^2 + N_i^2}, \text{ and } \tan\varepsilon_i = -\frac{N_i}{M_i} \tag{48}$$

In these expressions Σ_i includes only the terms the angles of which may be included in the same argument; that is, the angles which are multiples of the first. The value of k in (34) must be used, putting $n = \frac{1}{2}$ and taking only the inequality belonging to η_i.

Having obtained from observations m values of λ'_1 or λ'_2, or of both, corresponding to m values of the argument, the preceding equations give m conditions for determining, by the method of least squares, the value of the constants B_i and ε_i.

If we in like manner obtain the values of H_n for any values of η_i, we obtain from the first and third of (40), omitting the small terms just referred to (§ 24),

$$H'_1 - H_0 = A_0 + A_2 \text{ for high waters,}$$

and from the second and fourth of (40),

$$H'_2 - H_0 = A_0 - A_2 \text{ for low waters.}$$

But in this case we have (24),

$$A_0 = K_0 + K_0\, \Sigma_i\, R_i \cos(\eta_i - \alpha_i) = K_0 + K_0\, \Sigma_i\,(M_i \cos\eta_i + N_i \sin\eta_i)$$
$$A_2 = K_2 + K_2\, \Sigma_i\, R_i \cos(\eta_i - \alpha_1) = K_2 + K_2\, \Sigma_i\,(M_i \cos\eta_i + N_i \sin\eta_i)$$

in which Σ_i is limited as above, and in which

$$R_i = \sqrt{M_i^2 + N_i^2}, \text{ and } \tan\alpha_i = \frac{N_i}{M_i} \tag{49}$$

The values of R_i, M_i, &c., differ in their different connections with K_0 and K_2. From the preceding we get, for high waters,

$$\left\{\begin{array}{l} H'_1 - H'_0 - K_2 = K_0\, \Sigma_i\,(M_i \cos\eta_i + N_i \sin\eta_i) + K_2\, \Sigma_i\,(M_i \cos\eta_i + N_i \sin\eta_i) \\ H'_2 - H'_0 + K_2 = K_0\, \Sigma_i\,(M_i \cos\eta_i + N_i \sin\eta_i) - K_2\, \Sigma_i\,(M_i \cos\eta_i + N_i \sin\eta_i) \end{array}\right. \tag{50}$$

With m values of H'_1 and H'_2, corresponding to m values of the argument, we have, from the preceding, m equations of condition for determining the constants $R_{(s,\,i)}$ and $\alpha_{(s,\,i)}$.

From the first of (37) and from (41) we obtain, when λ_3 and λ_4 are reckoned from the lower transit,

$$\left\{\begin{array}{l} \tfrac{1}{2}(\lambda_1 - \lambda_3) = \sin q_1 \\ \tfrac{1}{2}(\lambda_2 - \lambda_4) = -\cos q_2 \end{array}\right. \tag{51}$$

when A_1 is small in comparison with A_2.

We also obtain from (40), in the case in which A_1 is not eliminated in the grouping of the observations,

$$\left\{\begin{array}{l} \tfrac{1}{2}(H_1 - H_3) = A_1 \cos\varDelta + A_3 \cos 3\varDelta \\ \tfrac{1}{2}(H_2 - H_4) = -A_1 \sin\varDelta + A_3 \sin 3\varDelta \end{array}\right. \tag{52}$$

In these equations A_3 is always so small that the values of K_3 and $\varDelta'$, obtained by the last two conditions of (42) for the constant and principal part of A_3, can be used without any sensible error.

With m values of λ_n for both high and low waters of both transits, (51) and (41) give m equa-

tions of condition for determining A_1 and Δ for m values of $\varphi=\frac{1}{2}\gamma_3$, when A_2 and A_3 and also Δ' have been determined from preceding conditions.

With m values of H_n, also, for both high and low waters of both transits, (52) gives, when A_3 and Δ' are known or insensible, m equations of condition for determining A_1 and Δ for m values of φ from observations of the heights only.

From (32) we have

$$A_1=K_1 \sin(\varphi-\alpha)=M \sin\varphi+N \cos\varphi \tag{53}$$

in which

$$K_1=\sqrt{M^2+N^2}, \text{ and } \tan\alpha=-\frac{N}{M} \tag{54}$$

With m values of A_1, determined by the preceding equations, (53) gives m equations of condition for determining K and ε.

From the first of (41) we obtain, approximately, when A_1 is small in comparison with A_2, and consequently $\sin q$ is small,

$$q_1=-\frac{A_1 \sin\Delta}{4A_2}$$

and from the second of (41),

$$q_2-\tfrac{1}{2}\pi=-\frac{A_1 \cos\Delta}{4A_2}$$

When q_1 and $q_2-\frac{1}{2}\pi$ are very small we can use for A_2 its mean value K_2, and then with the preceding value of A_1 we get

$$\left\{\begin{aligned} q_1&=-\frac{K_1 \sin\Delta}{4K_2}\sin(\varphi-\alpha)\\ q_2-\tfrac{1}{2}\pi&=-\frac{K_1 \cos\Delta}{4K_2}\sin(\varphi-\alpha)\end{aligned}\right. \tag{55}$$

The values of q_3 and q_4 are the complements of the preceding expressions respectively.

TABLES OF AVERAGE OR NORMAL VALUES.

26. The following tables contain the averages of groups of observations taken within certain limits of two arguments, and arranged, with reference to the averages of the arguments to which the observations correspond, in the form of tables of double entry. By summing these average results in two ways, the averages of all the observations contained within the limits of each group of either argument are obtained. The arguments have been for the most part divided into twenty-four equal parts, and the mean of the two limits of each division has been taken as the average of the values of the arguments, except in the case of $(\psi-\psi')$, in which the true average has been obtained. From these tabular results all the constants in this discussion have been obtained, and they might be treated in various other ways and many important results obtained which have not been brought out here.

An explanation of the notation contained in the headings of these tables may be found in sections (4), (8), and (11). The values of all the arguments except γ_2 are given for the time of the moon's apparent transit over the Washington meridian, happening a little more than two days before the time of high water, and which is the transit C according to Lubbock's notation. For the sake of convenience in grouping the observations, the values of γ_2 were used for a time two lunar days later, for which a reduction must be made when it is necessary to have the values of the arguments all referred to the same time.

To these tabular values a constant of 2 days must be added to the lunitidal intervals, and also 20 feet to the heights of high water, and 10 feet to those of low water, for the absolute heights above the assumed zero of the tide-gauge.

TABLE I—*Containing average values belonging to the Arguments* $(\psi - \psi') = \frac{1}{2}\eta_1$ *and* $\phi = \frac{1}{2}\eta_3$.

UPPER TRANSITS.

ϕ	$(\psi-\psi') = 0h.0m.$ .... $0h.30m.$						$(\psi-\psi') = 0h.30m.$ .... $1h.0m.$						$(\psi-\psi') = 1h.0m.$ .... $1h.30m.$					
	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2
°		h. m.	h. m.	h. m.	Ft.	Ft.		h. m.	h. m.	h. m.	Ft.	Ft.		h. m.	h. m.	h. m.	Ft.	Ft.
7.5	12	0 12	0 40	6 47	5.60	4.58	14	0 43	0 33	6 43	5.62	4.48	15	1 16	0 26	6 38	5.40	4.94
22.5	14	0 12	0 43	6 46	5.16	4.87	13	0 45	0 39	6 45	5.38	4.92	12	1 22	0 31	6 40	5.14	5.03
37.5	15	0 16	0 45	6 49	5.41	5.15	14	0 53	0 38	6 44	4.88	5.19	10	1 14	0 34	6 42	5.33	4.80
52.5	12	0 17	0 46	6 51	5.41	5.03	10	0 42	0 38	6 42	5.04	5.47	12	1 13	0 35	6 42	5.32	5.22
67.5	9	0 16	0 43	6 50	5.17	5.17	10	0 42	0 41	6 44	5.19	5.06	10	1 16	0 33	6 40	5.01	4.91
82.5	12	0 11	0 45	6 48	5.27	4.97	11	0 41	0 39	6 46	4.89	5.06	10	1 14	0 32	6 43	5.15	5.16
97.5	11	0 17	0 40	6 46	5.10	5.21	11	0 47	0 34	6 43	5.04	5.17	10	1 16	0 30	6 34	4.69	5.47
112.5	9	0 13	0 40	6 45	4.98	5.00	10	0 47	0 28	6 35	5.07	4.94	11	1 16	0 29	6 37	5.19	5.21
127.5	10	0 16	0 33	6 43	5.20	4.63	11	0 41	0 34	6 42	5.12	4.98	9	1 11	0 31	6 39	5.08	5.07
142.5	14	0 11	0 35	6 46	5.44	4.52	12	0 47	0 32	6 38	4.96	4.79	11	1 16	0 32	6 42	5.39	4.59
157.5	12	0 18	0 39	6 51	5.53	4.70	11	0 49	0 30	6 38	5.41	4.62	10	1 15	0 31	6 42	5.30	4.37
172.5	13	0 17	0 40	6 52	5.62	4.54	12	0 47	0 36	6 46	5.72	4.59	10	1 13	0 29	6 37	5.62	4.64
187.5	13	0 17	0 41	6 53	5.91	4.71	13	0 49	0 37	6 47	5.94	4.63	12	1 16	0 35	6 46	5.75	4.32
202.5	11	0 19	0 40	6 53	5.82	4.16	12	0 46	0 36	6 47	6.07	4.36	11	1 14	0 33	6 48	5.55	4.51
217.5	12	0 17	0 41	6 56	6.26	4.03	12	0 42	0 38	6 53	5.79	4.55	13	1 15	0 39	6 50	5.72	4.54
232.5	8	0 13	0 47	7 02	6.07	4.41	13	0 45	0 38	6 53	6.01	4.51	12	1 17	0 33	6 46	5.89	4.87
247.5	11	0 12	0 49	7 00	6.23	4.25	10	0 44	0 38	6 52	5.63	4.80	11	1 11	0 34	6 50	6.10	4.52
262.5	12	0 16	0 42	6 56	5.74	4.62	12	0 45	0 34	6 49	6.05	4.40	9	1 14	0 31	6 44	5.67	4.63
277.5	10	0 15	0 37	6 51	5.91	4.15	11	0 47	0 36	6 52	5.82	4.62	10	1 12	0 38	6 50	5.67	4.70
292.5	10	0 11	0 34	6 50	5.95	4.05	12	0 42	0 30	6 43	5.66	4.65	11	1 14	0 26	6 41	5.89	4.28
307.5	11	0 12	0 38	6 54	6.01	4.42	9	0 43	0 34	6 47	5.91	4.28	9	1 15	0 29	6 40	5.67	4.22
322.5	10	0 17	0 34	6 45	6.12	4.54	11	0 41	0 29	6 43	5.88	4.45	10	1 8	0 22	6 33	5.46	4.39
337.5	10	0 20	0 37	6 46	5.60	4.14	10	0 45	0 28	6 38	5.73	4.31	13	1 12	0 22	6 34	5.74	4.66
352.5	13	0 19	0 38	6 45	5.59	4.09	11	0 42	0 29	6 39	5.53	4.14	13	1 15	0 28	6 37	5.39	4.55
	273	0 15.2	0 40.3	6 50.3	5.63	4.58	275	0 44.8	0 34.6	6 44.5	5.51	4.71	264	1 14.4	0 30.9	6 41.4	5.46	4.73

ϕ	$(\psi-\psi') = 1h.30m.$ .... $2h.0m.$						$(\psi-\psi') = 2h.0m.$ .... $2h.30m.$						$(\psi-\psi') = 2h.30m.$ .... $3h.0m.$					
	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2
°		h. m.	h. m.	h. m.	Ft.	Ft.		h. m.	h. m.	h. m.	Ft.	Ft.		h. m.	h. m.	h. m.	Ft.	Ft.
7.5	12	1 51	0 26	6 35	5.10	4.84	10	2 11	0 22	6 31	5.00	5.24	11	2 41	0 21	6 31	5.34	5.51
22.5	11	1 46	0 25	6 34	5.08	4.65	12	2 16	0 25	6 34	4.93	5.26	13	2 45	0 25	6 35	4.97	5.24
37.5	9	1 40	0 33	6 41	5.42	4.94	13	2 13	0 26	6 33	4.84	5.76	14	2 47	0 23	6 33	4.73	5.56
52.5	14	1 44	0 29	6 40	4.71	5.31	14	2 20	0 28	6 37	4.70	5.70	12	2 48	0 23	6 30	4.52	5.62
67.5	11	1 45	0 26	6 35	4.79	5.51	12	2 16	0 26	6 34	4.80	5.90	12	2 41	0 24	6 33	4.44	5.65
82.5	12	1 43	0 31	6 35	5.13	5.68	12	2 17	0 26	6 33	4.74	5.69	12	2 49	0 19	6 27	4.44	5.97
97.5	10	1 43	0 32	6 37	4.56	5.24	10	2 15	0 25	6 34	5.34	5.50	10	2 43	0 23	6 30	4.87	5.94
112.5	12	1 46	0 27	6 38	4.98	5.38	9	2 16	0 20	6 28	4.74	5.58	10	2 41	0 16	6 25	4.59	5.38
127.5	11	1 46	0 20	6 28	5.04	5.04	11	2 16	0 16	6 25	4.72	5.21	10	2 43	0 19	6 29	4.95	5.38
142.5	10	1 42	0 20	6 28	5.29	4.88	12	2 13	0 21	6 26	4.87	5.02	10	2 46	0 18	6 30	4.76	5.23
157.5	12	1 43	0 31	6 38	5.10	4.91	12	2 11	0 25	6 34	4.95	4.91	9	2 40	0 23	6 32	5.16	5.79
172.5	13	1 42	0 28	6 38	5.29	4.82	12	2 17	0 23	6 33	5.06	5.05	12	2 51	0 25	6 33	5.06	5.21
187.5	14	1 43	0 26	6 39	5.52	4.56	13	2 15	0 27	6 40	5.18	5.03	12	2 44	0 25	6 37	5.31	5.23
202.5	12	1 45	0 30	6 42	5.54	4.90	14	2 19	0 28	6 40	5.16	4.98	12	2 48	0 22	6 35	5.12	5.15
217.5	13	1 48	0 27	6 41	5.50	4.70	11	2 20	0 28	6 38	5.52	5.11	11	2 42	0 23	6 40	5.47	5.09
232.5	14	1 43	0 30	6 49	5.58	5.00	14	2 12	0 30	6 41	5.48	5.09	12	2 43	0 25	6 42	5.32	4.93
247.5	12	1 44	0 31	6 44	5.67	4.92	14	2 15	0 28	6 39	5.34	5.29	13	2 45	0 23	6 37	5.44	5.33
262.5	11	1 44	0 27	6 43	5.61	5.17	12	2 16	0 21	6 35	5.71	5.15	10	2 40	0 23	6 41	5.59	5.24
277.5	11	1 44	0 25	6 38	5.55	4.85	10	2 14	0 21	6 37	5.77	4.94	11	2 44	0 21	6 36	5.19	5.38
292.5	12	1 44	0 17	6 33	5.64	4.46	12	2 14	0 24	6 30	5.77	4.55	11	2 52	0 18	6 34	5.19	4.80
307.5	11	1 45	0 21	6 35	5.63	4.83	11	2 19	0 15	6 38	5.63	5.03	11	2 44	0 17	6 30	5.17	5.02
322.5	11	1 42	0 24	6 38	5.73	4.70	12	2 14	0 26	6 38	5.46	4.86	11	2 43	0 15	6 27	5.19	5.08
337.5	12	1 45	0 22	6 33	5.57	4.68	10	2 20	0 23	6 36	5.71	4.85	10	2 47	0 13	6 27	5.35	5.22
352.5	12	1 49	0 25	6 36	5.42	4.92	11	2 15	0 20	6 33	5.44	4.92	10	2 42	0 13	6 25	5.08	5.32
	285	1 44.4	0 26.4	6 37.4	5.31	4.95	283	2 15.6	0 23.9	6 34.4	5.20	5.19	269	2 44.6	0 20.7	6 32.4	5.05	5.32

TABLE I—*Containing average values belonging to the Arguments* $(\psi - \psi') = \frac{1}{2}\eta_1$ *and* $\phi = \frac{1}{2}\eta_3$—Continued.

UPPER TRANSITS.

ϕ	$(\psi - \psi') = 3h.\ 0m.$ . . . . $3h.\ 30m.$						$(\psi - \psi') = 3h.\ 30m.$ . . . . $4h.\ 0m.$						$(\psi - \psi') = 4h.\ 0m.$ . . . . $4h.\ 30m.$					
	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H^2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2
°		h. m.	h. m.	h. m.	Ft.	Ft.		h. m.	h. m.	h. m.	Ft.	Ft.		h. m.	h. m.	h. m.	Ft.	Ft.
7.5	12	3 13	0 19	6 30	4.92	5.44	10	3 44	0 26	6 32	4.54	5.64	11	4 12	0 25	6 35	4.23	5.70
22.5	12	3 18	0 22	6 32	4.39	5.46	11	3 45	0 19	6 32	4.66	5.78	12	4 12	0 28	6 38	4.18	5.80
37.5	12	3 19	0 25	6 34	4.46	5.67	11	3 45	0 22	6 32	4.38	5.66	11	4 12	0 26	6 41	4.40	5.83
52.5	10	3 17	0 21	6 34	4.68	5.78	12	3 42	0 23	6 33	3.72	5.86	14	4 17	0 31	6 38	3.97	6.14
67.5	14	3 16	0 25	6 32	4.53	6.12	13	3 44	0 23	6 33	4.41	6.08	12	4 20	0 26	6 34	3.68	6.33
82.5	12	3 16	0 18	6 31	4.93	5.64	11	3 44	0 19	6 27	4.28	6.08	12	4 13	0 21	6 28	4.10	6.87
97.5	11	3 16	0 20	6 30	4.78	5.83	14	3 45	0 16	6 26	4.10	6.18	13	4 21	0 17	6 32	4.01	6.20
112.5	11	3 15	0 17	6 27	4.73	5.87	10	3 45	0 22	6 31	4.83	5.95	12	4 14	0 22	6 31	4.59	6.30
127.5	11	3 15	0 11	6 24	4.62	5.64	10	3 42	0 14	6 21	4.18	5.74	10	4 17	0 23	6 36	4.66	6.00
142.5	12	3 13	0 15	6 23	4.78	5.68	11	3 47	0 10	6 27	4.83	5.57	12	4 16	0 19	6 31	4.45	5.58
157.5	11	3 13	0 20	6 29	4.84	5.46	11	3 44	0 20	6 28	4.73	6.04	12	4 14	0 27	6 40	4.85	5.57
172.5	10	3 23	0 23	6 36	4.83	5.51	11	3 45	0 26	6 35	4.85	5.45	11	4 15	0 26	6 41	4.68	5.85
187.5	11	3 12	0 22	6 31	4.91	5.39	11	3 40	0 26	6 39	4.89	5.73	11	4 15	0 27	6 45	5.03	5.57
202.5	12	3 14	0 22	6 39	5.14	5.46	13	3 47	0 29	6 43	4.99	5.86	13	4 16	0 31	6 44	4.87	5.78
217.5	13	3 14	0 25	6 40	5.07	5.37	13	3 42	0 24	6 41	5.01	5.77	12	4 16	0 33	6 50	4.79	5.85
232.5	14	3 17	0 26	6 41	5.17	5.73	13	3 44	0 27	6 42	4.90	5.45	11	4 15	0 30	6 45	5.03	5.83
247.5	12	3 22	0 26	6 47	5.49	5.25	13	3 43	0 22	6 36	4.86	5.90	15	4 14	0 25	6 47	5.06	5.96
262.5	12	3 9	0 24	6 37	5.15	5.43	14	3 45	0 20	6 37	5.12	5.64	12	4 22	0 30	6 44	4.61	5.90
277.5	13	3 16	0 17	6 33	6.30	5.48	11	3 45	0 17	6 33	5.17	5.75	12	4 16	0 19	6 36	4.79	5.62
292.5	9	3 15	0 15	6 28	5.63	5.42	10	3 42	0 13	6 31	5.13	5.96	12	4 16	0 15	6 26	5.12	5.84
307.5	12	3 18	0 16	6 28	5.14	5.36	12	3 45	0 16	6 28	5.19	5.22	11	4 15	0 11	6 31	4.86	5.85
322.5	12	3 17	0 10	6 21	5.08	5.22	12	3 44	0 17	6 32	4.80	5.12	10	4 19	0 13	6 30	4.91	5.72
337.5	11	3 13	0 18	6 27	4.95	5.33	11	3 45	0 20	6 32	5.19	5.40	11	4 17	0 15	6 30	4.62	5.46
362.5	11	3 17	0 17	6 29	4.66	5.61	12	3 47	0 17	6 32	4.92	5.35	11	4 17	0 16	6 28	4.56	5.69
	280	3 15.8	0 19.8	6 31.8	4.92	5.55	280	3 44.2	0 20.3	6 32.6	4.74	5.71	283	4 15.9	0 23.2	6 36.7	4.58	5.88

ϕ	$(\psi - \psi') = 4h.\ 30m.$ . . . . $5h.\ 0m.$						$(\psi - \psi') = 5h.\ 0m.$ . . . . $5h.\ 30m.$						$(\psi - \psi') = 5h.\ 30m.$ . . . . $6h.\ 0m.$					
	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2
°		h. m.	h. m.	h. m.	Ft.	Ft.		h. m.	h. m.	h. m.	Ft.	Ft.		h. m.	h. m.	h. m.	Ft.	Ft.
7.5	12	4 45	0 31	6 43	4.43	5.68	11	5 15	0 33	6 45	4.21	5.72	10	5 44	0 42	6 53	4.27	5.72
22.5	11	4 43	0 35	6 46	4.60	5.66	12	5 16	0 36	6 49	4.10	6.20	10	5 42	0 38	6 52	4.33	5.91
37.5	16	4 46	0 31	6 42	4.03	6.05	11	5 18	0 41	6 53	4.21	6.28	10	5 46	0 44	6 53	4.34	5.90
52.5	12	4 47	0 32	6 42	3.89	6.02	11	5 14	0 40	6 51	4.03	5.95	12	5 39	0 46	6 54	3.78	6.43
67.5	12	4 46	0 30	6 42	3.93	5.88	12	5 16	0 39	6 47	3.68	6.37	13	5 43	0 46	6 58	3.62	5.93
82.5	14	4 43	0 27	6 37	4.27	6.43	13	5 17	0 35	6 45	3.92	6.49	13	5 46	0 38	6 50	3.70	6.04
97.5	11	4 47	0 20	6 28	4.00	6.33	12	5 16	0 27	6 42	4.43	6.43	14	5 43	0 36	6 46	4.01	6.57
112.5	12	4 42	0 20	6 30	4.61	5.91	12	5 16	0 26	6 40	4.05	6.55	13	5 46	0 29	6 44	4.35	6.33
127.5	12	4 46	0 19	6 28	4.32	6.53	11	5 18	0 23	6 37	4.24	6.22	11	5 42	0 29	6 41	4.38	6.12
142.5	11	4 45	0 20	6 31	4.43	6.15	11	5 17	0 28	6 42	4.72	5.89	12	5 41	0 32	6 46	4.40	6.20
157.5	12	4 46	0 20	6 35	4.31	6.06	10	5 13	0 29	6 44	4.74	5.62	11	5 43	0 34	6 49	4.73	6.01
172.5	9	4 45	0 31	6 45	4.48	5.49	11	5 16	0 29	6 46	4.70	5.94	11	5 45	0 40	6 57	4.89	5.90
187.5	11	4 44	0 32	6 43	4.47	5.59	11	5 15	0 32	6 51	4.61	5.57	11	5 44	0 40	6 56	5.03	5.94
202.5	11	4 47	0 40	6 52	4.85	5.41	10	5 14	0 41	6 58	4.67	5.80	11	5 46	0 48	6 59	4.83	5.77
217.5	13	4 42	0 36	6 50	4.69	5.76	12	5 17	0 41	7 0	4.56	5.40	12	5 44	0 51	7 8	5.06	6.14
232.5	12	4 47	0 30	6 51	4.84	5.85	14	5 16	0 48	7 5	4.75	6.01	12	5 47	0 47	7 7	4.86	5.82
247.5	13	4 44	0 33	6 49	4.58	6.09	12	5 18	0 35	6 58	4.97	5.90	14	5 45	0 47	7 12	4.70	6.42
262.5	12	4 39	0 22	6 41	4.87	6.19	13	5 12	0 37	6 55	4.71	5.91	15	5 40	0 35	6 55	4.95	6.15
277.5	15	4 45	0 24	6 44	4.87	5.84	14	5 19	0 32	6 51	4.58	6.07	12	5 47	0 36	6 56	5.05	6.31
292.5	11	4 45	0 16	6 34	4.79	6.02	12	5 12	0 21	6 41	4.80	5.99	13	5 44	0 33	6 52	5.43	5.84
307.5	10	4 44	0 17	6 32	4.84	5.72	12	5 16	0 25	6 42	4.69	5.83	13	5 45	0 27	6 44	4.51	6.11
322.5	10	4 45	0 19	6 35	4.79	5.68	12	5 16	0 19	6 36	4.52	5.82	11	5 50	0 28	6 43	4.29	5.72
337.5	11	4 47	0 22	6 39	4.72	5.75	12	5 13	0 29	6 45	4.69	5.78	11	5 44	0 29	6 43	4.35	6.13
352.5	10	4 46	0 28	6 41	4.45	5.85	12	5 13	0 30	6 44	5.05	5.57	13	5 47	0 36	6 49	4.34	5.93
	283	4 44.8	0 26.4	6 40.0	4.50	5.91	283	5 15.6	0 32.3	6 47.8	4.48	5.97	288	5 44.3	0 37.9	6 52.8	4.46	6.06

TABLE I—*Containing average values belonging to the Arguments* $(\psi - \psi') = \frac{1}{2}\eta_1$ and $\phi = \frac{1}{2}\eta_3$—Continued.

UPPER TRANSITS.

ϕ	$(\psi-\psi')=6h.\ 0m.\ \ldots\ 6h.\ 30m.$						$(\psi-\psi')=6h.\ 30m.\ \ldots\ 7h.\ 0m.$						$(\psi-\psi')=7h.\ 0m.\ \ldots\ 7h.\ 30m.$					
	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2
°		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
7.5	12	6 14	0 46	6 59	4.47	5.93	11	6 45	0 47	7 1	4.48	5.67	11	7 17	1 1	7 6	4.07	5.75
22.5	11	6 10	0 48	6 57	4.32	6.00	12	6 42	0 54	7 6	4.12	5.73	11	7 11	0 57	7 9	4.45	5.52
37.5	11	6 15	0 52	7 1	4.07	6.43	10	6 44	0 58	7 8	3.84	5.93	11	7 17	1 3	7 14	4.25	5.80
52.5	12	6 14	0 56	7 4	3.84	6.17	13	6 46	1 0	7 9	4.30	5.98	11	7 16	1 6	7 14	4.13	6.00
67.5	13	6 19	0 53	7 3	3.52	6.52	12	6 44	0 56	7 11	3.79	5.67	9	7 15	1 9	7 17	4.06	5.85
82.5	13	6 15	0 48	7 1	3.49	6.45	13	6 46	0 57	7 10	3.92	5.84	11	7 19	1 8	7 12	3.60	6.10
97.5	12	6 17	0 44	6 57	4.05	6.43	13	6 43	0 54	7 1	3.89	5.89	11	7 14	1 1	7 10	4.08	6.03
112.5	13	6 18	0 45	6 59	4.11	6.56	14	6 42	0 45	6 58	4.33	6.14	13	7 16	0 57	7 10	4.24	6.28
127.5	12	6 11	0 30	6 43	4.00	6.43	14	6 46	0 47	6 59	4.23	6.13	11	7 22	0 54	7 5	4.45	6.05
142.5	11	6 18	0 40	6 53	4.64	6.02	11	6 45	0 41	6 55	4.48	6.43	11	7 15	0 48	7 4	4.65	5.84
157.5	13	6 12	0 41	6 55	4.69	5.93	12	6 43	0 49	7 3	4.85	5.97	11	7 12	0 51	7 4	4.75	5.50
172.5	13	6 16	0 45	7 0	4.92	5.98	11	6 49	0 50	7 7	4.84	5.56	10	7 14	0 55	7 12	5.25	5.51
187.5	9	6 14	0 47	7 3	4.90	5.80	10	6 48	0 52	7 9	5.07	5.46	10	7 13	0 54	7 14	5.27	5.58
202.5	12	6 14	0 53	7 11	5.20	5.71	11	6 40	0 53	7 11	5.00	6.00	11	7 13	1 5	7 24	5.38	5.53
217.5	11	6 17	1 0	7 16	4.85	5.79	11	6 46	1 1	7 20	4.92	5.60	12	7 14	1 1	7 24	5.20	5.22
232.5	11	6 14	0 59	7 17	4.64	5.73	11	6 41	0 58	7 18	5.04	5.59	11	7 19	1 10	7 28	5.11	5.55
247.5	12	6 15	0 51	7 14	5.12	5.81	12	6 46	1 2	7 24	4.98	5.34	12	7 15	1 7	7 27	4.99	5.84
262.5	12	6 18	0 50	7 12	4.69	6.08	12	6 43	0 50	7 12	4.96	5.90	11	7 11	1 0	7 26	4.84	5.35
277.5	11	6 19	0 48	7 10	4.68	5.94	13	6 43	0 46	7 6	5.19	5.80	13	7 17	1 1	7 21	4.72	5.54
292.5	11	6 15	0 42	6 59	4.44	6.04	12	6 46	0 46	7 4	4.65	5.72	10	7 18	0 56	7 12	4.85	5.69
307.5	12	6 17	0 37	6 56	4.94	5.93	11	6 38	0 44	7 5	4.61	5.61	12	7 12	0 45	7 7	4.89	5.67
322.5	10	6 16	0 37	6 53	4.68	6.00	11	6 47	0 37	6 55	4.75	5.58	13	7 17	0 54	7 7	4.65	5.67
337.5	11	6 16	0 35	6 49	4.41	5.57	13	6 42	0 42	6 59	4.43	5.72	14	7 16	0 49	7 3	4.61	5.63
352.5	9	6 18	0 34	6 50	4.31	5.51	10	6 47	0 48	7 0	4.20	5.64	11	7 16	0 51	7 5	4.37	5.67
	276	6 15.5	0 45.9	7 0.9	4.46	6.03	283	6 44.2	0 50.8	7 6.3	4.54	5.79	271	7 15.4	0 58.4	7 13.1	4.62	5.72

ϕ	$(\psi-\psi')=7h.\ 30m.\ \ldots\ 8h.\ 0m.$						$(\psi-\psi')=8h.\ 0m.\ \ldots\ 8h.\ 30m.$						$(\psi-\psi')=8h.\ 30m.\ \ldots\ 9h.\ 0m.$					
	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2
°		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
7.5	10	7 49	0 59	7 11	4.32	5.60	11	8 19	1 6	7 14	4.41	5.19	13	8 45	1 9	7 14	5.00	5.13
22.5	12	7 47	1 1	7 13	4.65	5.54	11	8 15	1 7	7 15	4.53	5.54	11	8 48	1 4	7 14	5.44	5.24
37.5	10	7 43	1 8	7 19	4.27	5.64	10	8 13	1 11	7 19	4.12	5.54	11	8 44	1 5	7 15	4.71	5.36
52.5	11	7 47	1 6	7 16	4.44	5.71	9	8 16	1 15	7 19	4.47	5.85	10	8 43	1 8	7 15	4.78	5.21
67.5	11	7 43	1 3	7 12	4.35	5.85	11	8 15	1 7	7 17	4.61	5.53	11	8 42	1 11	7 15	4.47	5.33
82.5	11	7 44	1 4	7 17	4.24	5.60	13	8 12	1 16	7 22	4.08	5.87	13	8 44	1 1	7 11	4.62	5.41
97.5	12	7 43	1 1	7 13	4.34	5.53	13	8 16	1 11	7 18	3.99	5.54	12	8 47	1 7	7 14	4.50	5.17
112.5	14	7 43	0 59	7 10	4.28	5.72	12	8 20	1 3	7 14	4.34	5.66	13	8 44	1 6	7 14	4.67	5.13
127.5	12	7 45	0 59	7 9	4.60	5.66	13	8 12	1 0	7 11	4.64	5.51	14	8 43	1 1	7 11	4.59	5.10
132.5	14	7 40	0 54	7 5	4.82	5.58	14	8 17	1 4	7 15	4.73	5.38	14	8 46	1 3	7 12	5.04	5.08
157.5	11	7 43	0 58	7 12	5.14	5.94	12	8 17	1 2	7 14	5.09	5.20	11	8 46	0 58	7 9	5.29	5.01
172.5	12	7 45	0 59	7 15	4.73	5.56	10	8 20	1 2	7 17	5.43	5.06	10	8 44	1 9	7 26	5.64	5.32
187.5	13	7 45	1 0	7 21	5.31	5.19	12	8 16	1 5	7 21	5.26	5.06	9	8 47	1 8	7 21	5.76	5.02
202.5	12	7 41	1 3	7 21	5.34	5.26	12	8 16	1 3	7 23	5.52	4.81	11	8 43	1 12	7 27	5.67	4.94
217.5	10	7 47	1 7	7 27	5.45	5.48	11	8 14	1 10	7 26	5.59	4.97	10	8 43	1 2	7 19	6.08	4.43
242.5	12	7 47	1 9	7 28	5.63	5.08	11	8 19	1 13	7 31	5.35	4.95	11	8 47	1 13	7 31	5.58	5.05
247.5	10	7 42	1 9	7 31	5.13	5.01	12	8 13	1 13	7 27	5.28	5.51	11	8 45	1 10	7 27	6.21	4.90
262.5	14	7 41	1 9	7 26	5.09	5.08	12	8 17	1 9	7 31	5.24	5.12	10	8 46	1 2	7 21	5.78	5.14
277.5	13	7 41	1 1	7 20	4.93	5.44	11	8 17	1 3	7 23	5.15	4.76	13	8 43	1 7	7 25	5.40	4.52
292.5	13	7 46	0 58	7 18	5.26	5.30	13	8 17	1 3	7 18	5.12	5.25	12	8 46	1 9	7 24	5.51	4.66
307.5	14	7 42	0 52	7 13	5.07	5.70	14	8 18	1 3	7 19	4.93	5.15	13	8 45	0 56	7 12	5.65	4.78
322.5	11	7 46	0 58	7 12	4.95	5.48	11	8 16	0 57	7 9	5.05	5.73	12	8 41	0 56	7 9	5.68	5.07
337.5	13	7 46	0 51	7 8	4.77	5.29	12	8 12	0 59	7 11	5.06	5.30	13	8 44	0 58	7 13	5.25	5.36
352.5	9	7 46	0 57	7 9	4.91	5.44	11	8 12	1 0	7 11	4.97	5.36	12	8 43	1 2	7 12	4.94	5.26
	284	7 44.2	1 1.0	7 16.1	4.84	5.49	281	8 15.8	1 5.9	7 18.6	4.87	5.33	280	8 44.5	1 4.8	7 17.1	5.22	5.06

TABLE 1—*Containing average values belonging to the Arguments* $(\psi - \psi') = \frac{1}{2}\eta_1$ and $\phi = \frac{1}{2}\eta_3$—Continued.

UPPER TRANSITS.

ϕ	$(\psi-\psi')=9h.\ 0m.\ \ldots\ 9h.\ 30m.$						$(\psi-\psi')=9h.\ 30m.\ \ldots\ 10h.\ 0m.$						$(\psi-\psi')=10h.\ 0m.\ \ldots\ 10h.\ 30m.$					
	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2
°		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
7.5	11	9 18	0 58	7 5	5.25	4.56	11	9 43	1 1	7 11	5.65	5.01	12	10 11	1 2	7 7	5.26	5.18
22.5	11	9 17	1 5	7 13	4.92	5.05	10	9 44	1 2	7 9	5.18	4.98	12	10 13	0 59	7 9	5.10	5.06
37.5	10	9 18	1 11	7 14	4.78	5.07	10	9 40	1 5	7 13	4.58	5.06	12	10 11	1 3	7 10	5.39	5.24
52.5	11	9 13	1 16	7 21	4.63	5.41	10	9 44	1 8	7 14	5.01	5.27	14	10 16	1 2	7 8	4.84	5.42
67.5	12	9 15	1 8	7 9	4.68	5.46	12	9 46	1 6	7 10	4.83	5.12	11	10 17	1 4	7 11	5.10	5.21
82.5	11	9 16	1 12	7 15	4.85	5.36	10	9 46	1 3	7 5	4.68	4.83	10	10 15	1 0	7 5	4.83	5.19
97.5	10	9 13	1 6	7 14	4.54	5.34	11	9 42	0 55	7 5	5.08	5.02	12	10 14	1 1	7 5	5.00	5.20
112.5	12	9 17	1 3	7 12	4.44	5.04	12	9 47	0 59	7 7	4.90	5.12	11	10 17	0 58	7 3	4.74	5.08
127.5	10	9 19	1 6	7 14	4.61	5.46	12	9 46	1 1	7 6	5.26	4.56	12	10 12	0 56	7 5	5.04	4.78
142.5	12	9 17	0 58	7 6	5.28	5.08	12	9 42	1 0	7 9	5.32	4.72	13	10 12	0 59	7 6	5.25	5.11
157.5	15	9 14	1 2	7 14	5.36	4.88	13	9 43	0 57	7 11	5.52	4.90	12	10 20	0 56	7 5	5.47	4.92
172.5	13	9 11	0 59	7 16	5.55	4.65	14	9 47	1 0	7 13	5.54	5.03	14	10 17	0 58	7 9	5.65	4.23
187.5	12	9 12	1 5	7 19	5.73	4.86	11	9 45	1 9	7 21	5.75	4.77	12	10 17	1 0	7 16	6.03	4.19
202.5	12	9 14	1 5	7 17	5.78	5.02	11	9 50	1 6	7 24	5.94	4.90	10	10 16	1 0	7 14	6.04	4.57
217.5	12	9 12	1 13	7 27	5.36	4.96	12	9 46	1 8	7 23	5.93	4.48	10	10 16	1 2	7 15	6.21	4.32
232.5	12	9 16	1 6	7 22	6.12	4.36	12	9 48	1 5	7 21	5.97	4.74	10	10 17	1 5	7 18	6.02	4.61
247.5	10	9 13	1 12	7 32	5.29	4.97	11	9 43	1 3	7 22	6.23	4.10	12	10 15	1 1	7 16	5.90	4.45
262.5	10	9 16	1 3	7 25	5.83	4.64	11	9 44	1 4	7 23	5.72	4.70	10	10 15	1 1	7 18	5.51	4.71
277.5	11	9 16	1 4	7 25	5.27	4.76	11	9 48	0 58	7 13	5.91	4.94	8	10 14	0 56	7 12	6.09	5.09
292.5	11	9 17	1 2	7 18	5.74	4.42	11	9 41	1 0	7 14	5.63	4.30	11	10 13	0 55	7 8	5.63	4.39
307.5	12	9 12	0 57	7 16	5.53	4.77	12	9 41	0 59	7 14	5.98	4.27	13	10 16	0 55	7 9	5.63	4.37
322.5	16	9 13	1 1	7 12	5.16	4.65	15	9 46	0 57	7 10	5.76	4.84	12	10 16	0 53	7 10	5.95	4.57
337.5	14	9 19	1 3	7 12	5.27	5.01	12	9 52	0 52	7 4	5.43	4.49	11	10 19	0 57	7 10	5.73	4.43
352.5	12	9 18	1 1	7 12	5.08	4.89	10	9 44	0 58	7 7	5.60	4.86	11	10 12	0 55	7 5	5.63	4.97
	282	9 15.2	1 4.8	7 16.2	5.21	4.94	276	9 44.9	1 1.5	7 12.9	5.49	4.79	275	10 15.0	0 59.1	7 9.8	5.50	4.80

ϕ	$(\psi-\psi')=10h.\ 30m.\ \ldots\ 11h.\ 0m.$						$(\psi-\psi')=11h.\ 0m.\ \ldots\ 11h.\ 30m.$						$(\psi-\psi')=11h.\ 30m.\ \ldots\ 12h.\ 0m.$					
	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2
°		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
7.5	14	10 45	0 54	7 4	5.57	5.30	13	11 19	0 50	6 59	5.66	4.66	10	11 46	0 47	6 55	5.84	4.29
22.5	13	10 45	0 56	7 1	5.38	5.04	11	11 19	0 52	6 58	5.32	4.87	10	11 43	0 41	6 58	5.58	4.78
37.5	10	10 48	0 53	7 3	5.41	4.76	10	11 13	0 50	6 57	5.49	4.56	12	11 40	0 52	6 54	5.32	5.21
52.5	8	10 47	0 58	7 5	5.36	4.75	9	11 13	0 53	6 58	5.53	4.62	11	11 43	0 50	6 55	5.28	4.91
67.5	11	10 43	0 59	7 5	5.21	4.92	12	11 12	0 54	6 59	4.94	4.95	10	11 46	0 49	6 53	4.83	5.25
82.5	10	10 44	0 56	6 58	5.06	5.17	9	11 15	0 54	6 58	5.05	5.04	9	11 42	0 50	6 55	5.03	5.22
97.5	10	10 46	0 54	7 0	4.92	4.89	10	11 14	0 48	6 51	4.85	5.11	10	11 44	0 44	6 48	5.15	5.02
112.5	11	10 43	0 49	6 57	5.44	4.83	11	11 16	0 46	6 54	5.35	5.04	11	11 47	0 44	6 45	5.15	4.77
127.5	15	10 44	0 50	7 0	5.28	4.63	13	11 18	0 45	6 55	5.08	4.62	10	11 49	0 45	6 50	5.50	4.70
142.5	13	10 44	0 50	6 57	5.42	4.46	10	11 18	0 40	6 51	5.42	4.45	9	11 41	0 42	6 50	5.62	4.47
157.5	12	10 47	0 51	7 1	5.86	4.41	13	11 17	0 47	6 58	5.47	4.49	13	11 44	0 42	6 50	5.79	4.14
172.5	11	10 46	0 53	7 4	5.98	4.66	12	11 13	0 49	7 0	5.74	4.24	13	11 44	0 47	6 59	5.82	4.60
187.5	13	10 43	0 53	7 7	6.06	4.17	14	11 13	0 55	7 6	6.29	4.14	13	11 46	0 44	6 55	5.81	4.51
202.5	13	10 42	0 58	7 13	6.25	4.17	13	11 15	0 54	7 9	6.43	4.06	13	11 48	0 49	7 1	5.88	4.30
217.5	9	10 40	1 1	7 15	5.97	4.55	11	11 15	0 54	7 10	6.02	4.55	12	11 45	0 48	7 4	6.28	4.20
232.5	11	10 44	1 4	7 17	6.25	4.54	12	11 14	0 56	7 10	6.17	4.39	10	11 46	0 44	7 1	6.14	4.33
247.5	13	10 43	0 58	7 16	6.27	4.13	10	11 18	0 54	7 10	5.82	4.61	11	11 47	0 52	6 59	6.21	4.20
262.5	10	10 48	0 54	7 7	6.51	4.62	12	11 17	0 43	7 0	6.23	4.32	11	11 46	0 46	7 3	6.31	4.14
277.5	11	10 42	0 53	7 10	5.84	4.39	12	11 14	0 50	7 7	5.72	4.18	10	11 45	0 43	6 58	5.82	4.36
292.5	11	10 47	0 48	7 1	5.94	4.98	11	11 13	0 45	7 0	6.19	4.17	10	11 45	0 43	7 4	6.08	4.22
307.5	12	10 43	0 41	7 5	5.66	4.07	10	11 13	0 44	6 53	6.10	4.17	12	11 43	0 38	6 51	5.62	4.64
322.5	12	10 46	0 47	7 3	6.08	3.79	12	11 15	0 43	6 57	5.92	4.27	11	11 44	0 33	6 47	5.73	3.86
337.5	12	10 39	0 49	7 0	5.97	4.24	13	11 14	0 44	6 59	5.78	4.17	14	11 48	0 41	6 49	5.61	4.04
352.5	15	10 45	0 49	7 1	5.63	4.31	15	11 19	0 47	6 54	5.65	4.40	13	11 51	0 45	6 53	5.78	4.41
	280	10 44.3	0 53.7	7 4.3	5.72	4.58	278	11 15.3	0 49.0	6 59.7	5.68	4.50	268	11 45.1	0 45.4	6 55.3	5.67	4.58

TABLE II—*Containing average values belonging to the Arguments* $(\psi-\psi')=\frac{1}{2}\eta_1$ *and* $\phi=\frac{1}{2}\eta_3$.

LOWER TRANSITS.

ϕ	$(\psi-\psi')=0h.\ 0m.$ . . . . $0h.\ 30m.$						$(\psi-\psi')=0h.\ 30m.$ . . . . $1h.\ 0m.$						$(\psi-\psi')=1h.\ 0m.$ . . . . $1h.\ 30m.$					
	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4
°		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
7.5	14	0 17	0 40	6 50	5.67	4.60	13	0 48	0 33	6 46	5.87	4.05	10	1 15	0 28	6 39	5.91	4.32
22.5	11	0 22	0 38	6 51	6.11	4.09	11	0 47	0 33	6 47	6.00	4.28	13	1 11	0 32	6 44	5.90	4.31
37.5	8	0 17	0 40	6 52	5.95	4.31	12	0 41	0 34	6 48	6.01	4.51	15	1 15	0 30	6 44	5.86	4.58
52.5	13	0 11	0 39	6 54	6.11	4.05	13	0 46	0 35	6 47	6.08	4.45	8	1 17	0 30	6 45	6.02	4.81
67.5	10	0 20	0 38	6 54	6.23	4.43	11	0 51	0 39	6 50	5.88	4.37	10	1 14	0 24	6 42	5.72	4.30
82.5	9	0 14	0 42	6 58	5.95	4.49	10	0 43	0 33	6 50	6.19	4.18	10	1 14	0 26	6 48	5.82	4.52
97.5	11	0 17	0 38	6 51	6.31	4.39	10	0 44	0 34	6 50	6.18	4.40	11	1 14	0 30	6 50	5.62	4.55
112.5	12	0 30	0 34	6 48	6.10	4.10	11	0 44	0 27	6 43	6.07	4.50	11	1 16	0 27	6 38	5.59	4.61
127.5	10	0 20	0 33	6 47	5.57	4.30	10	0 48	0 28	6 42	6.13	4.25	10	1 15	0 19	6 31	6.01	4.36
142.5	11	0 18	0 38	6 52	5.93	4.53	10	0 44	0 33	6 47	5.49	4.64	10	1 12	0 29	6 42	5.58	4.80
157.5	12	0 16	0 34	6 47	5.92	4.19	12	0 44	0 30	6 43	5.86	4.31	13	1 14	0 33	6 46	4.99	4.66
172.5	13	0 12	0 43	6 53	5.88	4.62	13	0 42	0 35	6 46	5.60	4.54	12	1 15	0 27	6 39	5.57	4.82
187.5	13	0 12	0 44	6 51	5.56	4.76	14	0 43	0 38	6 48	5.21	4.85	14	1 16	0 33	6 43	5.40	4.78
202.5	12	0 13	0 46	6 53	5.51	4.98	12	0 47	0 38	6 48	5.32	4.70	12	1 18	0 31	6 44	5.04	5.12
217.5	15	0 14	0 48	6 56	5.11	5.08	13	0 47	0 42	6 50	5.43	5.27	12	1 14	0 37	6 43	5.02	5.25
232.5	12	0 22	0 48	6 56	5.38	5.14	10	0 49	0 42	6 51	5.26	5.33	9	1 10	0 39	6 48	5.29	4.92
247.5	11	0 18	0 43	6 50	5.38	5.07	10	0 41	0 49	6 52	5.22	5.60	12	1 12	0 41	6 49	4.87	5.57
262.5	11	0 18	0 49	6 51	5.20	5.25	11	0 48	0 44	6 49	5.03	5.54	11	1 15	0 35	6 43	5.20	5.74
277.5	10	0 13	0 42	6 52	4.69	5.25	10	0 40	0 41	6 44	4.93	5.35	11	1 16	0 32	6 41	5.07	5.67
292.5	11	0 15	0 38	6 53	5.03	5.20	11	0 44	0 30	6 43	5.03	5.01	11	1 11	0 30	6 36	5.21	4.99
307.5	11	0 17	0 45	6 44	5.13	5.33	11	0 45	0 36	6 38	5.24	4.55	11	1 16	0 28	6 37	4.95	5.09
322.5	12	0 13	0 36	6 45	5.66	4.20	11	0 45	0 23	6 35	4.95	4.84	10	1 16	0 34	6 40	5.16	4.93
337.5	14	0 10	0 36	6 46	5.27	4.39	13	0 46	0 34	6 43	5.67	4.60	9	1 19	0 30	6 39	5.02	4.94
352.5	14	0 16	0 37	6 46	5.85	4.07	13	0 49	0 31	6 41	5.39	4.45	8	1 18	0 29	6 44	6.13	4.64
	277	0 15.7	0 40.4	6 50.8	5.64	4.63	278	0 45.2	0 35.1	6 45.9	5.59	4.69	263	1 14.7	0 30.6	6 42.3	5.46	4.84

ϕ	$(\psi-\psi')=1h.\ 30m.$ . . . . $2h.\ 0m.$						$(\psi-\psi')=2h.\ 0m.$ . . . . $2h.\ 30m.$						$(\psi-\psi')=2h.\ 30m.$ . . . . $3h.\ 0m.$					
	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4
°		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
7.5	11	1 40	0 25	6 37	5.37	4.75	13	2 15	0 23	6 32	5.33	5.09	11	2 44	0 24	6 33	5.34	5.19
22.5	15	1 44	0 27	6 39	5.76	4.89	14	2 17	0 23	6 37	5.68	4.88	9	2 48	0 22	6 30	5.03	4.82
37.5	13	1 49	0 28	6 41	5.77	4.97	12	2 18	0 24	6 35	5.42	4.98	12	2 42	0 25	6 41	5.48	5.00
52.5	11	1 44	0 24	6 43	5.72	4.49	13	2 8	0 21	6 38	5.66	5.06	15	2 45	0 24	6 38	5.28	5.35
67.5	10	1 42	0 30	6 40	5.89	4.63	11	2 15	0 23	6 36	5.46	5.24	11	2 43	0 18	6 32	5.52	5.41
82.5	14	1 45	0 26	6 38	5.66	4.71	12	2 15	0 19	6 34	5.47	5.00	10	2 45	0 18	6 32	5.74	5.48
97.5	10	1 49	0 19	6 35	5.88	4.79	9	2 13	0 18	6 33	5.56	4.53	14	2 44	0 15	6 33	5.55	5.40
112.5	11	1 44	0 22	6 36	5.48	4.94	9	2 16	0 21	6 33	5.43	5.10	9	2 46	0 13	6 30	5.78	5.09
127.5	9	1 46	0 20	6 35	5.76	4.76	11	2 17	0 19	6 32	5.71	4.91	11	2 46	0 8	6 23	5.41	5.21
142.5	13	1 44	0 20	6 31	5.48	4.39	11	2 15	0 16	6 31	5.40	4.79	12	2 43	0 15	6 30	5.09	4.85
157.5	10	1 47	0 28	6 39	5.23	4.96	9	2 15	0 18	6 32	5.22	4.84	10	2 46	0 15	6 28	5.17	4.83
172.5	11	1 45	0 28	6 38	5.43	4.67	13	2 13	0 29	6 40	4.94	5.02	11	2 41	0 22	6 33	4.92	5.25
187.5	12	1 48	0 30	6 43	5.25	4.82	11	2 17	0 24	6 32	4.93	4.90	12	2 45	0 24	6 35	4.81	5.37
202.5	11	1 45	0 33	6 40	4.91	4.76	13	2 12	0 29	6 38	4.83	5.03	13	2 41	0 27	6 37	4.63	5.41
217.5	13	1 41	0 34	6 42	4.77	5.13	13	2 10	0 25	6 36	4.77	5.47	12	2 45	0 27	6 39	4.37	5.28
232.5	13	1 47	0 35	6 41	5.01	5.60	13	2 18	0 29	6 37	4.60	5.38	12	2 44	0 26	6 37	4.61	5.91
247.5	12	1 45	0 34	6 42	4.88	5.58	12	2 18	0 29	6 36	5.03	6.03	9	2 48	0 33	6 40	4.83	5.60
262.5	10	1 43	0 33	6 40	5.25	5.62	12	2 11	0 30	6 37	4.64	5.35	12	2 43	0 25	6 36	4.54	5.85
277.5	11	1 43	0 30	6 37	4.93	5.27	11	2 15	0 26	6 35	4.70	6.00	11	2 47	0 21	6 31	4.98	5.82
292.5	9	1 40	0 34	6 39	5.16	5.45	11	2 14	0 23	6 33	4.74	5.39	10	2 43	0 23	6 30	4.77	5.58
307.5	12	1 45	0 23	6 33	5.16	5.20	12	2 14	0 19	6 32	4.51	5.10	10	2 41	0 21	6 32	4.76	5.65
322.5	11	1 46	0 23	6 35	4.96	4.78	10	2 14	0 16	6 28	4.96	5.43	10	2 47	0 18	6 29	5.01	5.19
337.5	11	1 44	0 20	6 34	5.43	4.85	12	2 15	0 22	6 30	5.06	5.25	11	2 46	0 24	6 36	5.08	5.27
352.5	11	1 41	0 25	6 33	5.47	4.57	12	2 15	0 20	6 30	5.27	5.09	11	2 46	0 24	6 36	5.35	5.09
	274	1 44.4	0 27.1	6 37.9	5.36	4.94	279	2 14.6	0 22.8	6 34.0	5.14	5.17	268	2 44.6	0 21.3	6 33.4	5.09	5.33

TABLE II—*Containing average values belonging to the Arguments* $(\psi-\psi')=\frac{1}{2}\eta_1$ *and* $\phi=\frac{1}{2}\eta_3$—Continued.

LOWER TRANSITS.

ϕ	$(\psi-\psi')=3h.\ 0m.$ . . . . $3h.\ 30m.$						$(\psi-\psi')=3h.\ 30m.$ . . . . $4h.\ 0m.$						$(\psi-\psi')=4h.\ 0m.$ . . . . $4h.\ 30m.$					
	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4	Obs.	$(\psi-\psi')$	λ_3	λ_4	H'	H_4	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4
°		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
7.5	11	3 17	0 19	6 32	5.30	5.40	12	3 43	0 20	6 32	4.74	5.49	12	4 17	0 28	6 42	4.84	5.84
22.5	14	3 9	0 25	6 36	5.41	5.42	13	3 44	0 22	6 37	4.98	5.60	12	4 16	0 25	6 40	4.71	5.37
37.5	12	3 13	0 16	6 33	5.14	5.38	12	3 45	0 19	6 38	4.96	5.56	12	4 15	0 28	6 43	4.90	5.54
52.5	14	3 15	0 18	6 24	5.23	5.45	12	3 47	0 25	6 33	5.03	5.47	12	4 15	0 24	6 43	4.82	5.66
67.5	12	3 11	0 18	6 35	5.30	5.09	14	3 45	0 20	6 40	5.02	5.54	15	4 11	0 22	6 39	5.04	5.80
82.5	12	3 15	0 12	6 28	5.37	5.20	13	3 46	0 15	6 29	5.09	5.52	13	4 15	4 20	6 34	4.91	5.66
97.5	11	3 16	0 16	6 30	5.07	5.45	9	3 46	0 10	6 29	5.24	5.69	13	4 12	0 14	6 30	5.07	5.82
112.5	11	3 13	0 14	6 31	5.24	5.04	11	3 41	0 13	6 30	5.02	5.60	13	4 17	0 12	6 31	4.87	5.67
127.5	11	3 15	0 13	6 27	4.92	5.37	12	3 47	0 15	6 32	5.00	5.42	13	4 16	0 13	6 28	4.60	5.62
142.5	10	3 14	0 15	6 26	5.09	5.38	10	3 45	0 14	6 28	4.93	5.45	10	4 14	0 12	6 30	4.80	5.81
157.5	11	3 14	0 17	6 28	4.55	5.40	11	3 44	0 21	6 34	4.92	5.40	13	4 14	0 13	6 30	4.72	5.78
172.5	12	3 14	0 22	6 33	4.85	5.37	11	3 45	0 22	6 33	4.56	5.54	10	4 18	0 25	6 40	4.52	5.37
187.5	13	3 15	0 28	6 37	4.79	5.49	11	3 50	0 26	6 38	4.39	5.72	10	4 16	0 30	6 39	4.11	5.63
202.5	13	3 17	0 25	6 35	4.37	5.62	12	3 44	0 27	6 38	4.28	5.97	11	4 15	0 30	6 44	4.38	6.00
217.5	12	3 14	0 24	6 36	4.79	5.50	13	3 44	0 25	6 39	4.06	5.77	15	4 15	0 31	6 41	4.03	6.07
232.5	13	3 16	0 27	6 38	4.52	5.72	13	3 45	0 29	6 39	4.34	6.25	13	4 18	0 33	6 43	3.93	5.92
247.5	14	3 13	0 25	6 37	4.43	5.89	13	3 50	0 28	6 38	4.16	6.23	11	4 15	0 30	6 39	4.03	6.08
262.5	13	3 15	0 21	6 31	4.79	6.13	12	3 45	0 28	6 38	4.30	6.06	12	4 10	0 22	6 35	4.16	6.32
277.5	11	3 13	0 23	6 33	4.63	6.00	12	3 44	0 22	6 35	4.08	5.86	12	4 15	0 24	6 34	4.67	6.06
292.5	13	3 15	0 22	6 32	4.55	6.20	13	3 48	0 19	6 24	4.84	5.89	10	4 13	0 21	6 33	4.34	6.28
307.5	11	3 16	0 16	6 27	4.43	5.72	10	3 49	0 15	6 32	4.91	6.11	11	4 15	0 17	6 30	4.47	5.69
322.5	11	3 18	0 15	6 28	4.60	5.65	9	3 48	0 16	6 30	4.73	6.03	9	4 14	0 24	6 34	4.51	5.90
337.5	11	3 16	0 18	6 31	4.99	5.84	12	3 47	0 14	6 30	4.76	5.67	9	4 16	0 20	6 34	4.88	5.75
352.5	9	3 13	0 16	6 25	5.01	5.50	11	3 42	0 25	6 37	5.37	5.75	12	4 14	0 24	6 36	4.62	5.75
	285	3 14.5	0 19.4	6 31.4	4.89	5.55	281	3 45.6	0 20.3	6 33.9	4.74	5.73	283	4 14.8	0 22.6	6 36.3	4.58	5.81

ϕ	$(\psi-\psi')=4h.\ 30m.$ . . . . $5h.\ 0m.$						$(\psi-\psi')=5h.\ 0m.$ . . . . $5h.\ 30m.$						$(\psi-\psi')=5h.\ 30m.$ . . . . $6h.\ 0m.$					
	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4
°		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
7.5	11	4 44	0 27	6 44	4.83	5.75	10	5 12	0 35	6 48	4.78	5.69	10	5 44	0 39	6 53	4.78	5.86
22.5	7	4 48	0 33	6 48	5.00	5.90	11	5 17	0 39	6 50	4.85	5.85	11	5 46	0 45	7 1	4.82	5.79
37.5	13	4 43	0 32	6 52	4.64	6.02	11	5 13	0 34	6 52	4.93	5.70	12	5 44	0 44	7 5	4.79	6.07
52.5	13	4 44	0 33	6 52	4.81	5.73	13	5 14	0 36	6 52	4.49	5.78	10	5 46	0 42	7 1	5.06	5.71
67.5	13	4 45	0 25	6 53	4.61	6.22	12	5 15	0 34	6 56	4.91	5.62	13	5 46	0 42	7 1	4.64	5.76
82.5	12	4 44	0 22	6 40	4.77	5.98	12	5 14	0 30	6 51	4.87	5.75	9	5 46	0 35	6 54	4.64	6.22
97.5	12	4 49	0 21	6 38	4.87	5.93	12	5 16	0 16	6 37	4.91	6.10	10	5 44	0 32	6 51	4.76	5.98
112.5	11	4 50	0 17	6 32	4.42	6.25	11	5 15	0 18	6 39	4.72	5.73	11	5 44	0 26	6 43	4.82	6.29
127.5	10	4 48	0 20	6 35	4.90	5.68	11	5 13	0 18	6 39	4.56	5.77	11	5 45	0 28	6 46	4.44	6.03
142.5	11	4 44	0 22	6 37	4.46	5.80	12	5 14	0 20	6 35	4.72	5.89	12	5 47	0 32	6 49	4.58	5.88
157.5	11	4 46	0 19	6 36	4.84	5.53	10	5 16	0 27	6 49	4.60	6.07	12	5 49	0 32	6 46	4.49	5.89
172.5	12	4 47	0 32	6 42	4.23	5.47	10	5 16	0 33	6 48	4.36	6.00	11	5 44	0 34	6 48	4.15	5.61
187.5	10	4 46	0 32	6 42	4.19	5.79	11	5 13	0 41	6 51	3.90	5.85	12	5 42	0 41	6 56	4.07	5.74
202.5	11	4 44	0 32	7 44	3.84	5.89	9	5 17	0 48	6 57	4.06	5.95	10	5 49	0 49	7 0	3.96	6.00
217.5	11	4 49	0 40	6 55	3.89	6.33	11	5 17	0 44	6 59	4.20	6.27	9	5 44	0 50	6 50	3.90	5.76
232.5	12	4 47	0 40	6 50	3.76	6.50	11	5 9	0 40	6 52	3.99	6.10	13	5 41	0 56	7 4	3.61	6.49
247.5	11	4 45	0 37	6 45	4.05	6.18	15	5 12	0 45	6 55	3.61	6.37	14	5 47	0 48	7 3	3.76	6.32
262.5	12	4 51	0 31	6 45	4.30	6.65	13	5 12	0 36	6 46	4.05	6.42	13	5 43	0 49	6 58	3.97	6.13
277.5	12	4 45	0 30	6 43	4.10	6.30	11	5 18	0 28	6 40	4.18	6.25	11	5 48	0 46	6 53	3.56	6.46
292.5	13	4 43	0 22	6 34	4.05	6.25	13	5 14	0 28	6 43	4.30	6.21	11	5 43	0 33	6 50	4.35	6.28
307.5	12	4 44	0 20	6 36	4.53	6.00	10	5 12	0 28	6 43	4.30	6.43	11	5 44	0 34	6 50	4.12	6.21
322.5	10	4 42	0 20	6 34	4.69	5.92	11	5 13	0 25	6 39	4.40	5.90	13	5 45	0 32	6 47	4.42	5.97
337.5	12	4 46	0 27	6 42	4.59	5.98	12	5 17	0 33	6 46	5.05	6.00	11	5 45	0 31	6 48	4.62	5.87
352.5	10	4 44	0 27	6 43	5.08	5.60	11	5 15	0 32	6 48	4.67	5.94	11	5 43	0 38	6 53	5.00	5.75
	272	4 45.8	0 27.6	6 42.2	4.48	5.99	273	5 14.3	0 32.0	6 49.2	4.48	5.99	271	5 45.0	0 39.1	6 53.8	4.39	6.00

TABLE II—*Containing average values belonging to the Arguments* $(\psi-\psi')=\frac{1}{2}\eta_1$ *and* $\phi=\eta_3$—Continued.

LOWER TRANSITS.

ϕ	$(\psi-\psi')=6h.\ 0m.$ . . . . $6h.\ 30m.$						$(\psi-\psi')=6h.\ 30m.$ . . . . $7h.\ 0m.$						$(\psi-\psi')=7h.\ 0m.$ . . . . $7h.\ 30m.$					
	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_4	H_4	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4	Obs.	$(\psi-\psi')$	λ_2	λ_4	H_3	H_4
°		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
7.5	13	6 13	0 46	7 2	5.07	5.69	12	6 45	0 52	7 8	4.99	5.81	10	7 14	0 51	7 12	5.19	5.13
22.5	11	6 16	0 48	7 4	4.91	5.85	12	6 44	0 47	7 6	5.19	5.48	10	7 17	0 52	7 15	5.35	5.15
37.5	10	6 17	0 52	7 10	4.86	5.53	11	6 47	0 50	7 12	5.10	5.47	10	7 16	0 56	7 18	5.15	5.17
52.5	10	6 13	0 44	7 9	5.12	5.92	12	6 42	0 57	7 14	4.90	5.73	12	7 15	0 0	7 21	5.17	5.36
67.5	14	6 10	0 45	7 7	4.72	5.80	13	6 48	0 57	7 20	4.70	5.90	10	7 17	0 52	7 12	5.30	5.62
82.5	14	6 11	0 42	7 1	4.66	5.86	16	6 45	0 55	7 15	4.75	5.46	11	7 15	0 58	7 19	5.01	4.96
97.5	16	6 12	0 39	7 0	4.82	5.83	14	6 49	0 47	7 7	4.94	5.86	14	7 16	0 54	7 14	5.00	5.68
112.5	12	6 12	0 29	6 52	4.89	5.97	12	6 43	0 46	7 3	4.54	6.26	11	7 8	0 47	7 5	4.77	5.52
127.5	13	6 15	0 34	6 54	4.64	5.80	13	6 49	0 43	6 59	4.66	6.09	14	7 16	0 47	7 5	4.90	5.55
142.5	11	6 15	0 27	6 43	4.35	5.99	11	6 44	0 40	7 0	4.68	5.54	12	7 12	0 47	7 5	4.54	5.78
157.5	9	6 18	0 46	6 56	4.75	6.90	12	6 44	0 45	7 1	4.61	5.79	12	7 16	0 48	7 3	4.51	5.64
172.5	11	6 14	0 42	6 56	4.34	5.75	11	6 43	0 51	7 4	4.61	5.78	12	7 17	0 56	7 7	4.49	5.90
187.5	13	6 15	0 47	6 58	4.41	5.96	11	6 41	0 52	6 56	4.26	5.77	12	7 17	1 1	7 12	4.26	5.62
202.5	10	6 18	0 51	7 6	4.05	5.69	11	6 49	1 0	7 10	4.17	5.93	12	7 18	1 1	7 12	4.53	5.44
217.5	12	6 12	1 2	7 11	4.14	6.42	12	6 42	0 59	7 14	4.00	5.81	12	7 16	1 10	7 20	4.13	5.99
232.5	12	6 12	0 54	7 5	3.80	6.08	11	6 47	1 13	7 19	4.03	6.05	10	7 14	1 2	7 14	3.85	5.22
247.5	11	6 18	1 2	7 12	3.93	6.40	11	6 45	1 5	1 13	4.04	6.09	12	7 11	1 9	7 18	3.80	5.64
262.5	14	6 8	0 52	7 1	3.69	6.30	13	6 45	1 4	7 12	3.73	6.05	11	7 14	1 12	7 19	4.05	5.64
277.5	15	6 13	0 46	6 58	4.16	6.39	14	6 48	0 58	7 7	3.78	6.18	12	7 16	1 4	7 19	4.06	5.86
292.5	12	6 15	0 46	7 0	4.14	6.16	13	6 46	0 53	7 7	3.96	6.10	15	7 16	0 59	7 10	4.25	5.91
307.5	14	6 12	0 40	6 52	4.18	6.24	13	6 48	0 50	7 5	4.25	6.23	12	7 15	0 56	7 7	4.28	5.74
322.5	11	6 15	0 36	6 53	4.34	5.87	10	6 49	0 50	7 4	4.50	5.86	10	7 12	0 47	6 59	4.75	5.59
337.5	11	6 13	0 38	6 55	4.57	5.91	10	6 46	0 42	6 56	4.79	5.72	12	7 13	0 52	7 6	4.83	5.62
352.5	12	6 19	0 43	7 0	4.74	5.60	11	6 47	0 47	6 56	5.10	5.52	10	7 16	0 52	7 15	5.08	5.51
	291	6 14.0	0 44.6	7 0.2	4.47	5.96	289	6 46.0	0 52.6	7 7.0	4.51	5.85	278	7 14.8	0 56.4	7 11.6	4.64	5.55

ϕ	$(\psi-\psi')=7h.\ 30m.$ . . . . $8h.\ 0m.$						$(\psi-\psi')=8h.\ 0m.$ . . . . $8h.\ 30m.$						$(\psi-\psi')=8h.\ 30m.$ . . . . $9h.\ 0m.$					
	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4
°		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
7.5	10	7 41	0 57	7 13	5.37	5.36	11	8 17	1 2	7 19	5.26	5.08	10	8 44	0 56	7 12	5.82	4.53
22.5	11	7 47	1 4	7 21	5.31	5.35	11	8 17	1 3	7 19	5.64	4.88	11	8 45	1 6	7 18	5.31	4.83
37.5	10	7 45	1 1	7 24	5.69	5.16	12	8 14	1 6	7 22	5.69	5.37	9	8 48	1 4	7 21	6.25	4.46
52.5	9	7 43	1 2	7 27	4.93	5.40	12	8 17	1 2	7 24	5.71	4.47	12	8 46	1 7	7 30	5.46	5.23
67.5	9	7 44	1 2	7 25	5.57	5.33	11	8 13	1 1	7 22	5.57	4.93	9	8 45	1 2	7 21	5.23	5.05
82.5	11	7 42	1 5	7 30	5.18	5.57	11	8 14	1 3	7 24	5.32	5.26	11	8 44	1 9	7 25	5.65	5.10
97.5	13	7 47	1 0	7 19	5.02	4.93	13	8 14	1 2	7 21	5.16	5.21	10	8 44	1 3	7 19	5.18	4.80
112.5	13	7 44	0 55	7 15	4.79	5.45	15	8 12	0 57	7 13	5.27	4.97	12	8 49	1 1	7 20	5.17	4.80
127.5	12	7 49	0 54	7 9	4.83	5.90	13	8 17	0 56	7 12	5.14	4.97	13	8 48	0 58	7 18	5.15	5.01
142.5	12	7 46	0 52	7 10	4.94	5.50	13	8 15	0 57	7 11	5.21	5.41	12	8 44	1 2	7 17	5.07	5.43
157.5	12	7 45	0 59	7 11	4.87	5.63	11	8 12	0 58	7 12	4.61	5.66	13	8 48	0 56	7 8	5.13	5.18
172.5	11	7 47	0 58	7 9	4.95	5.28	12	8 15	1 7	7 12	5.00	5.61	13	8 43	1 3	7 15	5.08	5.28
187.5	11	7 44	1 0	7 13	4.73	5.53	11	8 13	1 9	7 16	4.78	5.62	13	8 46	1 12	7 18	4.82	5.34
202.5	9	7 47	1 13	7 21	4.45	5.50	12	8 12	1 9	7 18	4.55	5.60	12	8 44	1 11	7 16	4.73	5.32
217.5	11	7 43	1 12	7 21	4.46	5.74	11	8 18	1 12	7 20	4.49	5.40	12	8 49	1 17	7 24	4.62	5.44
232.5	11	7 46	1 17	7 24	4.17	5.72	11	8 14	1 17	7 23	4.36	5.79	11	8 46	1 16	7 24	4.26	5.52
247.5	12	7 44	1 16	7 22	4.24	6.00	10	8 16	1 10	7 16	4.31	5.48	12	8 46	1 12	7 21	4.51	5.46
262.5	11	7 48	1 9	7 22	4.35	5.67	12	8 13	1 14	7 21	4.09	5.40	10	8 44	1 13	7 22	4.64	5.51
277.5	11	7 47	1 11	7 20	3.67	5.74	11	8 15	1 9	7 19	4.14	5.52	13	8 49	1 15	7 19	4.14	5.40
292.5	13	7 46	1 5	7 12	4.48	5.84	13	8 13	1 7	7 19	4.58	5.57	13	8 43	1 5	7 13	4.44	5.29
307.5	11	7 45	1 3	7 13	4.63	5.49	13	8 9	1 9	7 16	4.65	5.53	13	8 46	1 6	7 14	4.99	5.20
322.5	15	7 45	0 57	7 11	4.88	5.77	14	8 15	1 0	7 13	4.86	5.42	11	8 48	1 1	7 13	4.79	4.87
337.5	13	7 43	0 51	7 5	5.10	5.36	13	8 17	0 57	7 13	5.00	5.24	10	8 46	0 55	7 12	5.37	4.64
352.5	12	7 46	0 57	7 12	4.97	5.21	11	8 17	1 1	7 15	5.37	5.00	10	8 46	0 56	7 15	5.59	4.81
	273	7 45.2	1 2.5	7 17.0	4.81	5.52	287	8 14.6	1 4.6	7 17.5	4.95	5.31	275	8 45.9	1 5.4	7 18.2	5.06	5.10

TABLE II.—*Containing average values belonging to the Arguments* $(\psi-\psi')=\frac{1}{2}\,\eta_1$ *and* $\phi=\frac{1}{2}\,\eta_3$—Continued.

LOWER TRANSITS.

ϕ	$(\psi-\psi')$ = 9h. 0m. . . . . 9h. 30m.					
	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4
°		h. m.	h. m.	h. m.	Ft.	Ft.
7.5	13	9 13	1 4	7 20	5.85	4.74
22.5	10	9 15	1 0	7 19	5.75	4.28
37.5	11	9 15	1 1	7 20	5.63	4.77
52.5	11	9 16	1 7	7 25	5.80	4.71
67.5	14	9 14	1 3	7 23	5.82	4.73
82.5	11	9 13	0 51	7 13	5.87	4.12
97.5	10	9 16	0 58	7 15	5.35	4.85
112.5	11	9 17	0 55	7 15	5.97	4.53
127.5	12	9 14	0 56	7 12	5.89	4.58
142.5	14	9 13	0 57	7 11	5.26	4.68
157.5	12	9 17	1 3	7 13	5.33	5.23
172.5	12	9 17	1 7	7 16	5.14	5.51
187.5	11	9 19	1 7	7 17	5.44	5.15
202.5	10	9 21	1 10	7 19	5.04	5.01
217.5	9	9 17	1 14	7 20	5.03	5.21
232.5	10	9 18	1 14	7 21	4.87	5.10
247.5	9	9 16	1 14	7 21	4.69	5.25
262.5	12	9 18	1 11	7 17	4.46	5.41
277.5	11	9 15	1 7	7 13	4.87	4.93
292.5	12	9 14	1 7	7 16	4.55	5.09
307.5	12	9 17	1 3	7 13	5.22	5.10
322.5	9	9 15	0 58	7 10	5.40	4.73
337.5	13	9 13	1 0	7 13	5.45	4.91
351.5	13	9 9	1 0	7 14	5.63	4.92
	272	9 15.5	1 3.6	7 16.5	5.34	4.90

ϕ	$(\psi-\psi')$ = 9h. 30m. . . . . 10h. 0m.					
	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4
°		h. m.	h. m.	h. m.	Ft.	Ft.
7.5	13	9 48	0 57	7 13	5.80	4.53
22.5	12	9 49	1 2	7 16	5.67	4.88
37.5	10	9 46	0 59	7 14	5.90	4.30
52.5	8	9 49	1 3	7 17	5.80	4.57
67.5	11	9 44	1 0	7 18	6.03	4.46
82.5	11	9 49	0 56	7 18	6.06	4.79
97.5	9	9 44	0 58	7 17	5.94	4.87
112.5	9	9 44	0 59	7 12	5.86	4.41
127.5	13	9 44	0 58	7 13	5.36	4.71
142.5	12	9 47	0 58	7 11	5.58	4.80
157.5	12	9 47	1 1	7 11	5.42	4.86
172.5	12	9 46	0 58	7 9	5.47	4.76
187.5	10	9 42	1 7	7 16	5.38	5.07
202.5	11	9 42	1 10	7 16	5.13	5.17
217.5	11	9 48	1 9	7 15	5.15	5.38
232.5	10	9 43	1 6	7 13	4.74	5.18
247.5	10	9 46	1 18	7 20	4.75	5.58
262.5	11	9 45	1 9	7 13	4.85	5.12
277.5	10	9 49	1 8	7 11	4.64	5.19
292.5	11	9 44	1 7	7 13	4.39	4.95
307.5	12	9 45	0 58	7 9	5.18	4.87
322.5	13	9 44	1 4	7 12	5.33	4.87
337.5	14	9 41	0 57	7 11	6.25	4.80
351.5	13	9 43	0 59	7 13	5.51	4.89
	268	9 55.4	1 2.6	7 13.8	5.42	4.88

ϕ	$(\psi-\psi')$ = 10h. 0m. . . . . 10h. 30m.					
	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4
°		h. m.	h. m.	h. m.	Ft.	Ft.
7.5	12	10 20	0 56	7 7	5.92	4.30
22.5	12	10 16	0 54	7 8	6.28	3.91
37.5	9	10 13	1 2	7 19	5.93	4.42
52.5	9	10 14	0 58	7 18	6.53	4.06
67.5	10	10 14	0 57	7 18	6.07	4.83
82.5	10	10 16	0 53	7 13	6.10	4.10
97.5	10	10 12	0 52	7 15	6.00	4.20
112.5	11	10 13	0 44	7 6	5.80	4.49
127.5	15	10 17	0 51	7 7	5.95	4.27
142.5	10	10 12	0 51	7 6	5.97	4.52
157.5	11	10 10	0 52	7 6	5.63	4.57
172.5	13	10 14	0 57	7 6	5.66	4.96
187.5	11	10 13	1 2	7 10	5.48	4.97
202.5	13	10 14	1 9	7 14	5.23	5.31
217.5	12	10 16	1 5	7 12	5.36	5.11
232.5	11	10 13	1 9	7 14	5.10	5.38
247.5	12	10 17	1 3	7 9	4.98	5.16
262.5	10	10 14	1 10	7 15	5.03	5.29
277.5	11	10 19	0 56	7 3	4.74	5.18
292.5	10	10 14	1 0	7 6	5.14	4.64
307.5	14	10 15	0 54	7 3	5.08	4.78
322.5	13	10 13	0 54	7 5	5.46	4.48
337.5	13	10 15	0 57	7 8	5.47	4.68
351.5	13	10 18	0 53	7 5	5.77	4.39
	276	10 14.7	0 57.4	7 9.7	5.61	4.67

ϕ	$(\psi-\psi')$ = 10h. 30m. . . . . 11h. 0m.					
	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4
°		h. m.	h. m.	h. m.	Ft.	Ft.
7.5	9	10 45	0 49	7 4	6.13	4.42
22.5	11	10 42	0 52	7 4	6.26	4.06
37.5	13	10 51	0 55	7 9	5.94	4.15
52.5	10	10 44	0 50	7 10	5.94	3.98
67.5	8	10 46	0 53	7 13	6.17	4.51
82.5	11	10 46	0 54	7 12	6.18	4.32
97.5	12	10 45	0 45	7 3	5.84	4.60
112.5	10	10 46	0 48	7 6	6.02	4.51
127.5	9	10 47	0 45	7 3	5.85	4.09
142.5	11	10 40	0 50	7 3	5.75	4.51
157.5	15	10 45	0 51	7 1	5.99	4.59
172.5	12	10 44	0 59	7 3	5.21	4.65
187.5	14	10 45	0 59	7 7	5.35	4.74
202.5	11	10 46	1 2	7 7	5.29	5.19
217.5	12	10 50	1 5	7 10	5.08	5.17
232.5	10	10 43	1 0	7 8	5.31	5.14
247.5	9	10 47	1 0	7 8	4.80	5.09
262.5	11	10 46	0 58	7 4	5.26	5.02
277.5	10	10 48	0 58	7 3	4.92	5.57
292.5	9	10 44	0 54	6 58	4.98	5.00
307.5	11	10 43	0 50	6 59	4.99	4.73
322.5	13	10 45	0 52	6 57	5.48	4.71
337.5	12	10 47	0 52	7 1	5.49	4.28
352.5	11	10 49	0 50	7 2	5.91	4.33
	264	10 45.5	0 53.8	7 4.8	5.59	4.64

ϕ	$(\psi-\psi')$ = 11h. 0m. . . . . 11h. 30m.					
	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4
°		h. m.	h. m.	h. m.	Ft.	Ft.
7.5	12	11 11	0 51	7 3	6.04	4.49
22.5	12	11 11	0 52	7 4	6.08	4.25
37.5	16	11 15	0 49	7 5	6.09	4.32
52.5	9	11 20	0 49	7 3	5.93	4.24
67.5	11	11 14	0 50	7 6	6.04	4.45
82.5	10	11 16	0 46	7 0	6.15	4.24
97.5	12	11 18	0 46	6 57	6.06	4.18
112.5	10	11 15	0 40	6 56	5.95	4.26
127.5	10	11 10	0 44	6 58	6.23	4.05
142.5	13	11 12	0 43	6 57	5.95	4.19
157.5	13	11 16	0 42	6 57	5.87	4.41
172.5	13	11 18	0 52	6 59	5.60	4.73
187.5	13	11 16	0 54	7 1	5.48	4.76
202.5	11	11 19	0 58	7 4	5.48	5.25
217.5	11	11 14	0 58	7 4	5.28	5.40
232.5	12	11 12	1 1	7 5	5.28	5.10
247.5	11	11 16	0 56	7 1	5.33	5.31
262.5	12	11 15	0 56	7 1	4.89	4.80
277.5	11	11 16	0 51	6 59	4.96	5.22
292.5	11	11 18	0 44	6 53	5.10	4.81
307.5	10	11 14	0 50	6 56	5.56	4.41
322.5	13	11 18	0 46	6 53	5.21	4.33
337.5	13	11 18	0 46	6 56	5.73	4.08
352.5	11	11 15	0 50	6 58	6.00	3.88
	280	11 15.1	0 47.2	6 59.8	5.68	4.55

ϕ	$(\psi-\psi')$ = 11h. 30m. . . . . 12h. 0m.					
	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4
°		h. m.	h. m.	h. m.	Ft.	Ft.
7.5	13	11 44	0 42	6 55	6.01	4.04
22.5	13	11 50	0 42	6 57	6.18	4.70
37.5	14	11 50	0 45	6 59	6.22	4.33
52.5	9	11 48	0 41	6 57	6.04	4.01
67.5	11	11 42	0 44	7 3	6.26	3.96
82.5	9	11 48	0 49	7 3	6.09	4.31
97.5	12	11 45	0 38	6 55	6.31	4.25
112.5	9	11 42	0 32	6 51	6.14	4.12
127.5	12	11 43	0 38	6 54	5.92	4.32
142.5	14	11 46	0 40	6 52	5.79	4.30
157.5	12	11 49	0 42	6 56	5.66	4.40
172.5	11	11 45	0 45	6 52	5.84	4.37
187.5	13	11 44	0 47	6 54	5.47	4.31
202.5	9	11 44	0 51	6 59	5.67	4.55
217.5	12	11 41	0 53	7 1	5.31	4.76
232.5	13	11 43	0 54	7 0	4.99	5.40
247.5	11	11 47	0 52	6 59	5.18	5.38
262.5	9	11 48	0 48	6 54	5.02	5.12
277.5	12	11 45	0 46	6 51	5.15	4.68
292.5	9	11 46	0 43	6 46	4.88	5.15
307.5	8	11 48	0 45	6 45	5.30	4.68
322.5	10	11 47	0 39	6 50	5.46	4.49
337.5	12	11 39	0 41	6 51	5.85	4.31
352.5	12	11 42	0 43	6 53	5.49	4.14
	269	11 45.8	0 44.2	6 54.8	5.68	4.50

TABLE III—*Containing average values belonging to the Arguments* $(\psi - \psi') = \frac{1}{2}\eta_1$ and η_2

COMBINED TRANSITS.

$(\psi-\psi')$	$\eta_2=15°$					$\eta_2=30°$					$\eta_2=45°$					$\eta_2=60°$				
	Obs.	λ'_1	λ'_2	H'_1	H'_2	Obs.	λ'_1	λ'_2	H'_1	H'_2	Obs.	λ'_1	λ_2'	H'_1	H'_2	Obs.	λ'_1	λ'_2	H'_2	H'_2
h. m.		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*
0 15	21	0 40	6 51	6.51	3.52	21	0 34	6 45	6.38	3.57	20	0 37	6 49	6.67	3.65	23	0 39	6 52	6.43	3.79
0 45	23	0 32	6 46	6.33	3.49	22	0 35	6 46	6.54	3.45	20	0 35	6 47	6.50	3.59	20	0 33	6 45	6.43	3.77
1 15	23	0 30	6 42	6.32	3.73	21	0 33	6 46	6.22	3.56	23	0 29	6 42	6.27	3.76	22	0 32	6 46	6.28	3.93
1 45	22	0 27	6 38	6.15	4.09	22	0 31	6 44	6.10	3.83	23	0 28	6 42	6.04	3.87	20	0 27	6 41	6.15	4.20
2 15	21	0 24	6 37	6.15	4.15	23	0 23	6 37	5.93	4.30	26	0 27	6 41	6.01	4.18	21	0 26	6 38	5.79	4.31
2 45	23	0 20	6 34	5.59	4.46	23	0 21	6 34	5/85	4.45	20	0 24	6 37	5.69	4.50	22	0 25	6 38	5.70	4.56
3 15	24	0 20	6 35	5.62	4.88	22	0 23	6 38	5.64	4.56	24	0 20	6 36	5.67	4.67	24	0 21	6 36	5.52	4.74
3 45	24	0 19	6 35	5.43	5.03	23	0 21	6 37	5.45	4.89	23	0 22	6 38	5.49	4.96	23	0 23	6 36	5.32	5.01
4 15	23	0 23	6 40	5.38	5.07	25	0 21	6 35	5.39	5.25	25	0 25	6 43	5.32	4.95	24	0 26	6 42	5.22	5.11
4 45	25	0 24	6 40	5.15	5.34	23	0 29	6 44	5.21	5.23	23	0 27	6 43	5.03	5.22	24	0 29	6 46	5.03	5.31
5 15	25	0 28	6 43	4.99	5.33	24	0 28	6 47	5.35	5.23	24	0 33	6 51	5.19	5.39	24	0 33	6 52	5.06	5.47
5 45	26	0 35	6 51	5.15	5.51	22	0 36	6 53	5.04	5.33	23	0 39	6 55	5.17	5.36	24	0 40	6 57	4.97	5.37
6 15	26	0 38	6 58	5.31	5.58	25	0 41	6 56	5.15	5.16	25	0 40	6 57	5.14	5.38	27	0 43	7 1	5.06	5.40
6 45	24	0 45	6 58	5.24	5.08	23	0 44	7 1	4.99	5.11	23	0 46	7 5	5 28	5.24	28	0 49	7 6	5.13	5.23
7 15	26	0 49	7 7	5.37	4.77	25	0 49	7 7	5.27	4.97	26	0 52	7 14	5.16	4.84	25	0 53	7 14	5.07	4.99
7 45	22	0 53	7 11	5.78	5.03	22	0 55	7 10	5.44	4.64	24	0 57	7 11	5.12	4.83	25	0 58	7 13	5.29	4.95
8 15	24	0 56	7 11	5.84	4.73	24	0 57	7 12	5.70	4.30	23	0 56	7 11	5.62	4.30	22	0 58	7 14	5.40	4.80
8 45	23	0 57	7 10	6.13	4.18	23	0 58	7 14	6.08	4.41	26	0 59	7 11	5.78	4.44	23	0 58	7 10	5.60	4.58
9 15	23	0 56	7 8	6.21	4.12	22	0 58	7 11	6.22	4.18	22	0 57	7 12	6.14	4.11	22	0 59	7 14	6.00	4.07
9 45	22	0 54	7 6	6.64	3.88	23	0 54	7 9	6.51	3.87	22	0 58	7 11	6.33	4.08	21	0 58	7 9	6.07	4.22
10 15	24	0 52	7 4	6.46	3.87	22	0 52	7 5	6.45	3.79	22	0 54	7 6	6.40	3.90	22	0 52	7 7	6.53	3.98
10 45	25	0 47	7 0	6.59	3.46	21	0 47	7 1	6.79	3.82	24	0 50	7 3	6.85	3.78	19	0 52	7 7	6.47	3.84
11 15	21	0 44	6 56	6.61	3.47	23	0 45	6 59	6.70	3.39	23	0 46	6 57	6.74	3.58	22	0 42	7 2	6.62	3.89
11 45	21	0 37	6 50	6.57	3.48	22	0 42	6 56	6.78	3.32	21	0 41	6 54	6.80	3.56	25	0 43	6 57	6.37	3.76
	561	0 37.9	6 52.3	5 89.4	4 42.6	546	0 39.0	6 53.6	5.882	4.378	555	0 40.1	6 54.8	5.850	4.427	552	0 40.8	6 56.0	5.771	4.554

$(\psi-\psi')$	$\eta_2=75$					$\eta_2=90°$					$\eta_2=105°$					$\eta_2=120°$				
	Obs.	λ'_1	λ'_2	H'_1	H'_2	Obs.	λ'_1	λ'_2	H'_1	H'_2	Obs.	λ'_1	λ'_2	H'_1	H'_2	Obs.	λ'_2	λ'_2	H'_1	H'_2
h. m.		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*
0 15	21	0 42	6 52	6.31	4.13	23	0 38	6 50	6.05	4.10	21	0 37	6 48	5.74	4.21	22	0 40	6 52	5.53	4.82
0 45	23	0 35	6 48	6.23	4.01	21	0 38	6 44	5.92	4.49	20	0 33	6 45	5.43	4.30	23	0 33	6 44	5.55	4.64
1 15	22	0 31	6 43	6.18	4.14	20	0 32	6 46	5.88	4.42	23	0 30	6 42	5.64	4.59	18	0 32	6 43	5.25	4.59
1 45	24	0 28	6 41	5.90	4.28	22	0 28	6 39	5.78	4.45	22	0 26	6 36	5.44	4.89	22	0 28	6 41	5.33	5.06
2 15	22	0 25	6 38	5.80	4.42	22	0 26	6 49	5.66	4.85	24	0 28	6 39	5.40	4.99	24	0 27	6 39	5.34	5.35
2 45	25	0 21	6 36	5.58	4.72	22	0 24	6 34	5.50	4.83	22	0 23	6 36	5.48	5.25	17	0 24	6 35	5.16	5.42
3 15	20	0 22	6 36	5.37	5.04	23	0 25	6 37	5.25	5.01	23	0 22	6 38	5.31	5.26	22	0 21	6 34	5.09	5.69
3 45	23	0 22	6 40	5.33	4.99	22	0 24	6 42	5.09	5.39	23	0 24	6 37	4.98	5.51	24	0 23	6 37	4.97	5.84
4 15	22	0 26	6 40	5.09	5.35	20	0 28	6 41	5.00	5.45	24	0 28	6 44	4.79	5.31	23	0 27	6 40	4.74	5.93
4 45	25	0 29	6 44	4.83	5.32	23	0 32	6 48	4.88	5.77	22	0 33	6 48	4.69	5.84	21	0 32	6 44	4.56	6.04
5 15	22	0 35	6 51	4.67	5.52	27	0 34	6 50	4.75	5.68	25	0 36	6 51	4.71	5.83	24	0 36	6 52	4.47	5.97
5 45	24	0 41	7 0	4.93	5.50	23	0 44	6 59	4.83	5.67	21	0 42	6 58	4.45	5.96	22	0 46	7 2	4.49	6.11
6 15	22	0 48	7 3	4.80	5.48	23	0 48	7 5	4.71	6.00	26	0 48	7 4	4.67	5.96	24	0 52	7 6	4.41	6.15
6 45	24	0 42	7 9	5.03	5.22	25	0 53	7 8	4.73	5.54	22	0 57	7 12	4.81	5.84	23	0 55	7 13	4.63	5.99
7 15	26	0 57	7 14	4.95	5.12	24	0 58	7 15	4.75	5.38	26	0 58	7 14	4.64	5.52	25	1 6	7 19	4.62	5.88
7 45	25	0 57	7 12	5.21	4.99	22	1 2	7 20	5.13	5.17	25	1 4	7 19	4.68	5.54	24	1 5	7 21	4.83	5.65
8 15	24	1 1	7 18	5.46	4.84	24	1 4	7 21	5.04	4.90	25	1 7	7 21	4.93	5.17	22	1 9	7 22	4.61	5.33
8 45	26	1 2	7 16	5.40	4.97	22	1 6	7 18	5.30	4.90	26	1 5	7 18	4.98	5.04	25	1 10	7 22	4.87	5.23
9 15	24	1 0	7 12	5.79	4.53	24	1 5	7 17	5.43	4.84	23	1 7	7 19	5.16	4.92	23	1 7	7 18	4.95	5.21
9 45	23	0 56	7 10	5.73	4.18	23	1 0	7 12	5.77	4.67	22	1 3	7 15	5.48	4.62	24	1 3	7 18	5.21	4.92
10 15	21	0 55	7 8	6.23	3.91	25	0 54	7 8	5.80	4.38	24	0 58	7 6	5.58	4.68	22	1 1	7 13	5.04	4.88
10 45	21	0 50	7 2	6.14	3.94	23	0 51	7 3	5.86	3.99	22	0 52	7 4	5.66	4.53	24	0 56	7 7	5.59	4.90
11 15	21	0 47	6 58	6.30	4.03	22	0 47	6 57	6.01	4.11	25	0 51	7 2	5.83	4.56	24	0 51	7 2	5.44	4.65
11 45	21	0 45	6 57	6.33	3.99	23	0 41	6 54	5.90	4.03	23	0 44	6 54	5.90	4.36	23	0 46	6 58	5.69	4.69
	551	0 42.0	6 56.2	5 56.6	4 69.2	548	0 43.4	6 57.0	5.376	4.917	559	0 44.0	6 57.1	5.182	5.112	565	0 45.4	6 58.4	5.015	5.372

TABLE III—*Containing average values belonging to the Arguments* $(\psi-\psi')=\eta_1$ *and* η_2—Continued.

COMBINED TRANSITS.

$(\psi-\psi')$	$\eta_2=135°$ Obs.	λ'_1	λ'_2	H'_1	H'_2	$\eta_2=150°$ Obs.	λ'_1	λ'_2	H'_1	H'_2
h. m.		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*
0 15	24	0 45	6 54	5.40	4.96	23	0 42	6 55	5.05	5.30
0 45	24	0 38	6 49	5.22	4.98	21	0 38	6 46	5.16	5.29
1 15	24	0 35	6 46	5.21	5.03	24	0 33	6 45	5.03	5.40
1 45	22	0 29	6 40	5.30	5.12	22	0 30	6 42	5.06	5.22
2 15	21	0 26	6 37	5.04	5.41	23	0 29	6 36	4.71	5.47
2 45	24	0 23	6 35	4.87	5.44	20	0 22	6 33	4.85	5.64
3 15	20	0 26	6 39	4.76	5.89	23	0 24	6 33	4.54	5.77
3 45	22	9 26	6 38	4.66	5.85	22	0 21	6 36	4.55	6.04
4 15	22	0 26	6 41	4.62	5.92	24	0 27	6 39	4.29	6.16
4 45	24	0 32	6 47	4.41	6.09	21	0 36	6 44	4.29	6.29
5 15	22	0 36	6 53	4.32	6.21	23	0 42	6 45	4.15	6.45
5 45	21	0 44	6 56	4.23	6.18	23	0 43	6 59	4.09	6.39
6 15	23	0 53	7 9	4.27	6.07	24	0 50	7 5	4.20	6.26
6 45	23	1 6	7 15	4.37	5.88	24	0 54	7 9	4.01	6.15
7 15	21	1 3	7 22	4.46	6.11	24	0 8	7 23	4.09	6.05
7 45	25	1 9	7 24	4.74	5.92	22	1 13	7 26	4.35	6.00
8 15	26	1 14	7 26	4.74	5.81	25	1 13	7 26	4.53	5.84
8 45	25	1 10	7 25	4.86	5.48	22	1 14	7 25	4.52	5.41
9 15	24	1 14	7 25	4.94	5.37	25	1 10	7 23	4.93	5.38
9 45	24	1 8	7 18	5.18	5.32	24	1 11	7 20	4.98	5.38
10 15	25	1 1	7 12	5.20	5.15	24	1 6	7 16	4.88	5.28
10 45	23	0 58	7 8	5.09	4.85	24	1 1	7 9	5.08	5.15
11 15	22	0 53	7 2	5.38	5.22	24	0 55	7 5	5.09	5.00
11 45	23	0 45	6 56	5.23	4.98	22	0 50	6 58	5.22	5.28
	554	0 47.5	6 59.9	4.854	5.552	553	0 47.8	6 59.5	4.652	5.692

$(\psi-\psi')$	$\eta_2=165°$ Obs.	λ'_1	λ'_2	H'_1	H'_2	$\eta_2=180°$ Obs.	λ'_1	λ'_2	H'_1	H'_2
h. m.		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*
0 15	22	0 44	6 53	5.07	5.45	23	0 44	6 52	4.95	5.39
0 45	24	0 39	6 43	5.03	5.35	22	0 37	6 45	4.82	5.31
1 15	20	0 36	6 43	4.99	5.46	21	0 32	6 41	4.65	5.43
1 45	25	0 31	6 39	4.73	5.65	24	0 33	6 41	4.54	5.66
2 15	20	0 24	6 31	4.44	5.73	21	0 23	6 36	4.68	5.96
2 45	23	0 24	6 34	4.74	6.16	23	0 24	6 33	4.65	6.30
3 15	20	0 23	6 32	4.54	6.01	20	0 20	6 33	4.35	6.25
3 45	22	0 25	6 36	4.36	6.22	21	0 18	6 28	4.13	6.32
4 15	23	0 28	6 40	3.88	6.15	24	0 30	6 41	4.11	6.50
4 45	20	0 34	6 46	3.88	6.34	23	0 33	6 44	4.08	6.71
5 15	20	0 34	6 49	4.16	6.47	23	0 37	6 51	3.98	6.59
5 45	20	0 46	7 0	3.99	6.71	21	0 47	6 59	3.87	6.50
6 15	23	0 53	7 5	4.07	6.61	22	0 52	7 8	3.90	6.69
6 45	25	1 0	7 15	4.07	6.32	23	1 0	7 14	4.00	6.49
7 15	21	1 1	7 16	4.04	6.35	24	1 10	7 25	4.14	6.23
7 45	25	1 11	7 26	4.29	6.12	23	1 14	7 27	4.11	6.24
8 15	26	1 17	7 30	4.32	5.79	22	1 17	7 28	4.32	6.11
8 45	25	1 17	7 23	4.23	5.67	26	1 17	7 28	4.36	5.90
9 15	22	1 14	7 25	4.62	5.50	23	1 16	7 26	4.66	5.60
9 45	25	1 10	7 20	4.84	5.46	23	1 12	7 23	4.77	5.70
10 15	23	1 4	7 14	4.89	5.36	25	1 7	7 17	4.94	5.49
10 45	24	1 0	7 10	4.90	5.04	24	1 1	7 10	5.02	5.37
11 15	24	0 54	7 4	5.10	5.13	25	0 55	7 3	4.87	5.22
11 45	24	0 51	7 0	5.18	5.12	24	0 50	6 56	4.85	5.23
	546	0 48.3	6 58.9	4.515	5.840	550	0 48.7	6 59.5	4.448	5.966

$(\psi-\psi')$	$\eta_2=195°$ Obs.	λ'_1	λ'_2	H'_1	H'_2	$\eta_2=210°$ Obs.	λ'_1	λ'_2	H'_1	H'_2
h. m.		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*
0 15	24	0 47	6 56	4.80	5.37	25	0 42	6 52	4.71	5.50
0 45	23	0 39	6 47	4.75	5.55	24	0 39	6 47	4.84	5.48
1 15	23	0 32	6 41	4.68	5.58	23	0 37	6 45	4.76	5.74
1 45	21	0 27	6 36	4.63	5.72	25	0 27	6 38	4.70	5.80
2 15	25	0 25	6 34	4.32	5.99	22	0 25	6 35	4.37	5.91
2 45	19	0 23	6 33	4.47	6.08	24	0 22	6 31	4.36	6.13
3 15	22	0 25	6 32	3.99	6.31	22	0 20	6 32	4.30	6.39
3 45	22	0 21	6 33	4.44	6.56	22	0 20	6 29	3.99	6.38
4 15	22	0 23	6 36	3.96	6.45	25	0 22	6 33	3.87	6.63
4 45	20	0 30	6 38	3.91	6.62	24	0 27	6 41	3.90	6.50
5 15	21	0 37	6 53	3.79	6.89	19	0 30	6 43	3.86	6.74
5 45	19	0 45	6 58	4.04	6.88	24	0 43	6 56	3.74	6.65
6 15	22	0 52	7 8	3.96	6.72	20	0 52	7 4	3.90	6.83
6 45	24	1 2	7 15	3.88	6.71	23	0 56	7 12	3.95	6.49
7 15	22	1 6	7 19	3.88	6.08	22	1 7	7 18	4.05	6.43
7 45	22	1 11	7 23	4.02	6.24	23	1 13	7 28	4.39	6.19
8 15	25	1 16	7 30	4.27	5.89	20	1 16	7 27	4.23	6.16
8 45	25	1 19	7 29	4.45	5.97	23	1 16	7 28	4.26	5.97
9 15	22	1 16	7 25	4.57	5.81	26	1 12	7 23	4.51	5.70
9 45	25	1 13	7 24	4.72	5.67	22	1 13	7 22	4.80	5.80
10 15	24	1 10	7 18	4.89	5.45	22	1 8	7 18	4.75	5.54
10 45	24	1 5	7 15	4.92	5.54	25	1 4	7 13	4.83	5.41
11 15	24	0 55	7 4	4.83	5.27	25	0 59	7 8	5.07	5.43
11 45	25	0 50	6 57	4.82	5.39	24	0 49	6 59	4.96	5.34
	545	0 48.7	6 59.3	4.374	6.014	555	0 47.5	6 58.4	4.379	6.047

$(\psi-\psi')$	$\eta_2=225°$ Obs.	λ'_1	λ'_2	H'_1	H'_2	$\eta_2=240°$ Obs.	λ'_1	λ'_2	H'_1	H'_2
h. m.		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*
0 15	24	0 47	6 54	5.05	5.60	22	0 41	6 51	5.03	5.45
0 45	27	0 33	6 45	4.79	5.58	23	0 40	6 49	4.86	5.54
1 15	25	0 31	6 40	4.86	5.64	24	0 30	6 38	4.89	5.78
1 45	22	0 27	6 37	4.49	5.75	24	0 25	6 33	4.52	5.58
2 15	23	0 22	6 32	4.60	5.99	23	0 23	6 32	4.48	5.97
2 45	22	0 21	6 32	4.30	6.03	26	0 20	6 31	4.28	5.92
3 15	22	0 20	6 30	4.28	6.21	25	0 20	6 29	4.13	6.18
3 45	21	0 22	6 33	3.99	6.54	23	0 23	6 34	4.27	6.38
4 15	23	0 21	6 35	4.02	6.50	25	0 18	6 32	3.95	6.36
4 45	21	0 27	6 37	3.57	6.78	25	0 28	6 38	3.81	6.59
5 15	25	0 35	6 48	3.81	6.57	20	0 32	6 46	3.75	6.65
5 45	18	0 37	6 53	3.81	6.66	19	0 40	6 53	3.68	6.49
6 15	23	0 47	7 1	3.77	6.71	22	0 44	7 0	3.74	6.52
6 45	22	0 57	7 10	3.79	6.61	20	0 56	7 9	3.97	6.66
7 15	22	1 4	7 17	3.91	6.44	22	1 0	7 11	4.02	6.31
7 45	19	1 6	7 21	4.21	6.10	23	1 8	7 20	4.16	6.17
8 15	25	1 16	7 24	4.11	5.77	23	1 11	7 20	4.33	5.86
8 45	22	1 12	7 25	4.44	5.72	24	1 8	7 19	4.49	5.76
9 15	21	1 12	7 23	4.56	5.66	22	1 13	7 22	4.48	5.67
9 45	23	1 9	7 19	4.91	5.53	22	1 9	7 19	4.92	5.48
10 15	25	1 6	7 14	4.91	5.68	22	1 4	7 11	4.74	5.46
10 45	20	0 59	7 7	4.84	5.51	25	0 59	7 9	5.18	5.27
11 15	26	0 56	7 6	4.98	5.42	26	0 52	7 1	4.90	5.46
11 45	26	0 52	6 59	4.96	5.60	22	0 48	6 57	5.16	5.32
	547	0 45.8	6 56.7	4.398	6.025	552	0 44.7	6 55.2	4.406	5.953

TABLE III—*Containing average values belonging to the Arguments* $\psi - \psi' = \frac{1}{2}\eta_1$ *and* η_2—Continued.

COMBINED TRANSITS.

$\psi - \psi'$	$\eta_2 = 255°$					$\eta_2 = 270°$					$\eta_2 = 285°$					$\eta_2 = 300°$				
	Obs.	λ'_1	λ'_2	H'_1	H'_2	Obs.	λ'_1	λ'_2	H'_1	H'_2	Obs.	λ'_1	λ'_2	H'_1	H'_2	Obs.	λ'_1	λ'_2	H'_1	H'_2
h. m.		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*
0 15	24	0 43	6 53	5.14	5.25	25	0 41	6 48	5.23	4.99	22	0 35	6 48	5 17	4 76	21	0 38	6 49	5.43	4.63
0 45	23	0 36	6 46	4.79	5.32	23	0 33	6 42	5.18	5.09	24	0 32	6 43	5.17	5.12	24	0 31	6 42	5.47	4.70
1 15	24	0 32	6 41	5.00	5.44	24	0 30	6 40	5.04	5.25	23	0 24	6 36	5.24	5.09	21	0 27	6 37	5.52	4.90
1 45	24	0 26	6 36	4.85	5.59	25	0 23	6 32	4.97	5.39	23	0 23	6 32	5.01	5.20	24	0 23	6 34	5.39	5.17
2 15	23	0 23	6 33	4.64	5.70	27	0 22	6 30	4.72	5.65	22	0 19	6 29	5.04	5.46	25	0 21	6 30	5.07	5.33
2 45	24	0 18	6 28	4.49	5.96	24	0 21	6 31	4.64	5.89	25	0 20	6 30	5.08	5.86	21	0 14	6 29	5.18	5.46
3 15	24	0 18	6 27	4.12	5.99	25	0 12	6 24	4.51	5.96	25	0 15	6 26	4.55	5.92	23	0 15	6 26	4.86	5.86
3 45	25	0 16	6 30	4.10	6 15	24	0 18	6 29	4.15	6.18	26	0 15	6 26	4.56	5.94	26	0 16	6 31	4.65	5.79
4 15	24	0 20	6 33	3.96	6.47	25	0 19	6 31	4.17	6.19	23	0 18	6 31	4.35	6.09	25	0 18	6 29	4.33	5.95
4 45	24	0 24	6 34	4.00	6.44	23	0 25	6 40	4.04	6.45	24	0 20	6 33	4.25	6.23	22	0 19	6 34	4.34	5.90
5 15	24	0 31	6 42	3.83	6.46	24	0 26	6 41	4.02	6.35	25	0 30	6 45	4.10	6.15	25	0 25	6 42	4.41	6.08
5 45	24	0 37	6 51	3.82	6.63	22	0 34	6 48	4.19	6.45	24	0 36	6 50	4.25	6.34	24	0 31	6 45	4.32	6.02
6 15	21	0 40	6 56	3.73	6.41	25	0 43	6 57	4.09	6.43	21	0 37	6 54	4.22	6.23	23	0 40	6 54	4.29	5.88
6 45	22	0 52	7 6	3.94	6.38	25	0 49	7 2	4.18	6.25	23	0 47	7 1	4.23	6.08	23	0 47	7 1	4.72	5.91
7 15	22	0 59	7 15	4.31	6.03	22	0 56	7 7	4.13	6.28	23	0 48	7 5	4.40	5.70	26	0 48	7 5	4.54	5.72
7 45	19	1 1	7 14	4.30	6.14	20	1 0	7 18	4.51	5.72	22	0 58	7 9	4.63	5.64	22	0 53	7 6	4.74	5.26
8 15	21	1 9	7 21	4.63	5.94	22	1 5	7 17	4.76	5.70	22	1 1	7 14	4.74	5.50	22	0 59	7 10	4.98	5.15
8 45	22	1 9	7 20	4.67	5.62	22	1 4	7 17	4.76	5.58	25	1 4	7 15	5.01	5.41	19	0 56	7 15	5.09	5.05
9 15	22	1 4	7 14	4.64	5.44	24	1 5	7 16	5.12	5.39	20	1 5	7 13	5.27	5.06	22	1 0	7 12	5.25	4.88
9 45	22	1 1	7 12	4.76	5.38	22	1 0	7 10	5.18	5.12	22	0 59	7 11	5.26	5.02	23	0 59	7 9	5.56	4.62
10 15	22	1 2	7 11	5.07	5.00	21	0 56	7 6	5.20	5.32	21	0 55	7 5	5.41	4.82	23	0 55	7 6	5.62	4.64
10 45	21	0 55	7 5	4.94	5.13	22	0 55	7 4	5.28	5.13	22	0 51	7 1	5.44	4.97	24	0 50	7 0	5.74	4.60
11 15	23	0 51	7 0	5.08	5.10	23	0 48	6 58	3.14	4.66	23	0 46	6 55	5.45	4.86	21	0 46	6 58	5.88	4.53
11 45	25	0 47	6 56	5.24	5.15	19	0 42	6 50	5.02	4.70	23	0 41	6 51	5.34	4 80	21	0 40	6 51	5.70	4.71
	549	0 42.3	6.535	4.502	5.797	556	0 40.3	6 51.6	4.676	5.630	553	0 38.3	6 50.1	4.840	5.510	550	0 37.1	6 49.8	5.045	5.281

$\psi - \psi'$	$\eta_2 = 315°$					$\eta_2 = 330°$					$\eta_2 = 345°$					$\eta_2 = 360°$				
	Obs.	λ'_1	λ'_2	H'_1	H'_2	Obs.	λ'_1	λ'_2	H'_1	H'_2	Obs.	λ'_1	λ'_2	H'_1	H'_2	Obs.	λ'_1	λ_2	H'_1	H'_2
h. m.		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*Ft.*	*Ft.*
0 15	23	0 35	6 48	5.73	4.54	20	0 34	6 46	6.13	4.05	24	0 36	6 46	6.15	3.82	22	0 37	6 50	6.32	3.56
0 45	22	0 30	6 40	5.77	4.30	20	0 32	6 41	5.99	4.27	24	0 32	6 44	6.36	4.02	21	0 35	6 47	6.31	3.79
1 15	25	0 26	6 39	5.46	4.39	20	0 26	6 38	5.84	4.53	27	0 29	6 41	5.99	4.21	20	0 31	6 43	6.12	3.79
1 45	23	0 24	6 36	5.37	4.63	23	0 25	6 37	5.84	4.52	24	0 24	6 36	5.86	4.51	23	0 26	6 37	6.16	4.23
2 15	23	0 18	6 30	5.40	5.10	25	0 18	6 32	5.42	4.77	24	0 20	6 33	5.80	4.68	23	0 23	6 36	6.02	4.49
2 45	23	0 18	6 28	5.10	5.35	23	0 19	6 30	5.12	5.03	22	0 18	6 32	5.07	4.45	24	0 22	6 37	5.81	4.70
3 15	23	0 15	6 26	4.89	5.45	24	0 15	6 29	5.22	5.45	21	0 14	6 25	5.49	5.05	24	0 21	6 34	5.58	4.93
3 45	21	0 14	6 29	5.03	5.59	25	0 17	6 28	4.84	5.52	22	0 18	6 33	5.32	5.53	25	0 18	6 31	5.16	5.00
4 15	24	0 17	6 30	4.64	5.63	26	0 16	6 30	4.91	5.52	24	0 21	6 35	5.01	5.33	25	0 20	6 33	5.08	5.21
4 45	26	0 21	6 36	4.38	5.87	22	0 22	6 35	4.56	5.62	25	0 23	6 38	4.83	5 58	22	0 25	6 38	5.00	5.50
5 15	26	0 24	6 41	4.54	5.91	23	0 25	6 41	4.65	5.65	27	0 26	6 41	4.84	5.43	24	0 29	6 44	4.91	5.54
5 45	23	0 31	6 45	4.49	5.70	23	0 33	6 50	4.64	5.80	25	0 30	6 46	4.88	5.52	26	0 32	6 48	4.97	5.47
6 15	24	0 37	6 53	4.47	5.75	23	0 38	6 52	4.73	5.42	22	0 38	6 45	4.99	5.40	23	0 38	6 55	5.24	5.24
6 45	22	0 46	7 1	4.62	5.55	24	0 41	6 58	4.86	5.41	24	0 43	6 59	4.94	5.28	21	0 43	6 58	5.28	5.08
7 15	24	0 47	7 1	4.81	5.34	24	0 47	7 5	5.03	5.16	23	0 47	7 2	5.18	5.14	23	0 44	7 2	5.37	5.07
7 45	24	0 53	7 9	5.12	5.37	23	0 53	7 5	5.22	4.91	23	0 51	7 8	5.51	4.92	25	0 53	7 8	5.46	4.86
8 15	24	0 56	7 11	5.23	4.90	22	0 55	7 6	5.31	4.78	24	0 51	7 6	5.51	4.45	24	0 55	7 9	5.71	4.50
8 45	23	0 55	7 8	5.30	4.71	24	0 56	7 12	5.66	4.46	21	0 55	7 7	5.67	4.40	22	0 56	7 8	5.91	4.26
9 15	21	0 55	7 8	5.55	4.36	23	0 53	7 7	5.62	4.36	22	0 53	7 7	5.86	3.98	22	0 53	7 6	6.00	4.09
9 45	19	0 54	7 7	5.80	4.36	23	0 52	7 4	5.92	4.15	23	0 52	7 5	6.21	3.88	22	0 53	7 5	6.20	3.78
10 15	21	0 54	7 5	5.56	4.35	20	0 48	7 2	6.04	3.87	23	0 49	7 2	6.11	3.89	22	0 47	7 3	6.58	3.78
10 45	22	0 46	7 0	6.06	4.03	22	0 43	7 0	6.07	4.16	20	0 46	7 0	6.40	3.56	22	0 45	7 0	6.62	3.74
11 15	22	0 45	6 55	6.04	4.40	22	0 41	6 53	6.07	3.87	20	0 43	6 55	6.24	3.60	23	0 46	6 58	6.41	3.59
11 45	24	0 41	6 50	5.81	4.22	22	0 41	6 53	6.10	3.96	22	0 38	6 49	6.31	3.61	23	0 41	6 54	6.61	3.31
	554	0 35.9	6 49.0	5.215	4.992	546	0 35.4	6 48.9	5.408	4 80.2	556	0 35.7	6 49.4	5.605	4.593	551	0 37.2	6 51.0	5.772	4.480

TABLE IV—*Containing average values belonging to the Arguments $\phi' = \frac{1}{2}\eta_5$ and Ω, or to the month and year.*

COMBINED TRANSITS.

Year.	JANUARY.				FEBRUARY.				MARCH.			
	λ'_1	λ'_2	H'_1	H'_2	λ'_1	λ'_2	H'_1	H'_2	λ'_1	λ'_2	H'_1	H'_2
	h. m.	*h. m.*	*Ft.*	*Ft.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
1848	0 38	6 52	4.87	5.03	0 41	6 55	5.25	5.29	0 38	6 53	5.05	5.04
1849	0 33	6 46	4.89	4.83	0 41	6 55	5.17	4.87	0 40	6 54	5.14	4.86
1850	0 38	6 54	5.37	5.27	0 39	6 54	5.10	5.06	0 41	6 55	5.16	4.99
1851	0 40	6 54	5.01	4.92	0 40	6 53	4.81	4.83	0 43	6 56	5.18	5.05
1852	0 41	6 54	5.00	5.44	0 36	6 50	4.83	4.94	0 40	6 55	4.93	4.01
1853	0 31	6 46	5.57	5.67	0 38	6 53	5.19	5.15	0 32	6 45	5.07	4.88
1854	0 33	6 47	4.89	4.97	0 37	6 51	4.70	5.24	0 35	6 49	4.78	4.78
1855	0 35	6 48	4.97	5.28	0 39	6 58	5.29	5.16	0 44	6 56	4.96	5.01
1856	0 35	6 48	5.14	5.63	0 31	6 44	4.81	5.18	0 36	6 50	4.77	5.01
1857	0 34	6 46	5.21	5.43	0 42	6 54	4.69	5.25	0 40	6 53	4.88	4.88
1858	0 39	6 53	4.74	5.33	0 48	6 52	4.36	5.12	0 41	6 54	4.75	4.92
1859	0 34	6 50	4.69	5.24	0 35	6 51	4.89	5.31	0 40	6 54	4.90	5.44
1860	0 36	6 50	4.53	5.21	0 40	6 53	4.59	4.96	0 39	6 54	4.74	5.05
1861	0 38	6 53	5.01	5.30	0 41	6 55	4.61	4.94	0 38	6 52	4.98	5.04
1862	0 42	6 54	4.88	5.12	0 37	6 48	5.02	5.46	0 41	6 53	5.37	5.47
1863	0 42	6 52	4.64	5.14	0 40	6 49	4.54	4.89	0 45	6 55	4.66	5.11
1864	0 44	6 56	4.23	4.90	0 47	6 57	4.48	4.66	0 45	6 57	5.17	5.56
1865	0 39	6 42	5.21	4.96	0 53	6 47	5.21	5.12	0 44	6 54	5.12	4.95
1866	0 49	7 3	4.98	4.95	0 59	6 55	4.44	4.61	0 43	6 55	4.66	5.21
Mean	0 37.9	6 51.0	4.933	5.190	0 40.7	6 52.3	4.841	5.060	0 40.8	6 53.4	4.962	5.058

Year.	APRIL.				MAY.				JUNE.			
	λ'_1	λ'_2	H'_1	H'_2	λ'_1	λ'_2	H'_1	H'_2	λ'_1	λ'_2	H'_1	H'_2
	h. m.	*h. m.*	*Ft.*	*Ft.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
1848	0 41	6 54	5.05	4.90	0 43	6 55	5.39	5.08	0 45	6 57	5.45	5.17
1849	0 44	6 56	4.92	4.70	0 46	6 58	5.10	5.07	0 50	7 0	5.39	5.41
1850	0 40	6 53	5.12	4.88	0 42	6 54	5.06	5.21	0 48	6 58	5.69	5.12
1851	0 46	6 57	5.31	5.33	0 43	6 54	4.98	5.09	0 45	6 54	5.19	5.43
1852	0 41	6 54	5.63	5.39	0 42	6 54	5.13	4.99	0 45	6 58	4.99	5.17
1853	0 39	6 52	5.00	4.97	0 42	6 54	5.05	5.16	0 45	6 56	5.02	5.07
1854	0 41	6 54	4.99	5.03	0 43	6 55	4.76	5.16	0 46	6 59	5.01	5.47
1855	0 42	6 55	4.81	4.95	0 42	6 54	5.15	5.46	0 42	6 56	4.93	5.34
1856	0 40	6 54	4.79	5.19	0 41	6 53	5.00	5.43	0 44	6 56	4.90	5.43
1857	0 39	6 51	5.08	5.36	0 43	6 57	4.89	5.24	0 49	7 1	5.10	5.45
1858	0 34	6 56	4.72	5.34	0 46	6 57	4.72	5.36	0 46	6 56	4.82	5.34
1859	0 39	6 51	4.89	6.31	0 40	6 52	4 88	5.11	0 41	6 51	4.83	4.93
1860	0 38	6 53	4.75	5.06	0 45	6 56	4.97	5.06	0 47	6 58	5.19	4.35
1861	0 44	6 56	5.36	5.42	0 42	6 54	5.18	5.34	0 45	6 56	4.98	5.28
1862	0 43	6 56	4.85	5.11	0 46	6 59	4.92	5.30	0 46	6 59	5.18	5.42
1863	0 37	6 48	4.96	5.36	0 45	6 55	5.21	5.46	0 49	6 58	5.20	5.34
1864	0 45	6 57	5.79	5.69	0 42	6 56	5.37	5.35	0 43	6 58	5.13	5.12
1865	0 46	6 59	5.14	5.12	0 47	7 0	5.46	5.58	0 51	7 6	5.19	5.25
1866	0 38	6 52	4.85	5.25	0 44	6 57	5.21	5.35	0 47	7 1	5.12	5.18
Mean	0 41.4	6 54.1	5.000	5.177	0 43.4	6 55.5	5.075	5.253	0 46.0	6 57.8	5.090	5.225

TABLE IV—*Containing average values belonging to the Arguments* $\phi' = \frac{1}{2}\eta_5$ *and* Ω, *or to the month and year*—Continued.

COMBINED TRANSITS.

Year.	JULY.				AUGUST.				SEPTEMBER.			
	λ'_1	λ'_2	H'_1	H'_2	λ'_1	λ'_2	H'_1	H'_2	λ'_1	λ'_2	H'_1	H'_2
	h. m.	*h. m.*	*Ft.*	*Ft.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
1847	0 43	6 57	5.28	5.06	0 44	6 56	5.35	4.93	0 46	6 58	5.40	5.35
1848	0 49	7 1	5.32	5.19	0 48	6 59	5.29	5.14	0 46	6 59	5.28	5.50
1849	0 54	7 3	5.09	5.24	0 54	7 5	5.39	5.43	0 48	6 59	5.27	5.37
1850	0 45	6 55	5.20	5.22	0 48	6 56	5.38	5.35	0 46	6 57	5.37	5.35
1851	0 45	6 57	5.37	5.36	0 45	6 55	5.33	5.18	0 46	6 57	5.23	5.22
1852	0 47	6 59	4.95	5.11	0 48	7 0	5.09	5.23	0 48	6 58	5.35	5.49
1853	0 47	6 58	5.04	5.43	0 48	6 57	5.16	5.86	0 42	6 55	5.07	5.63
1854	0 44	6 55	4.87	5.28	0 43	6 54	5.00	5.39	0 43	6 55	5.03	5.52
1855	0 45	6 57	5.09	5.28	0 42	6 54	5.15	5.42	0 42	6 55	5.17	5.80
1856	0 46	6 54	5.05	5.56	0 44	6 57	5.29	5.91	0 42	6 53	5.47	5.73
1857	0 53	6 59	5.07	5.56	0 40	6 51	4.76	5.90	0 43	6 53	4.90	5.74
1858	0 46	6 56	5.02	5.45	0 46	6 54	5.03	5.33	0 43	6 53	4.87	5.33
1859	0 42	6 54	5.04	5.01	0 42	6 51	5.12	4.99	0 41	6 52	4.98	5.16
1860	0 49	7 2	4.94	5.24	0 44	6 56	4.96	5.17	0 45	6 57	4.75	5.05
1861	0 49	7 1	4.92	5.23	0 48	7 1	4.88	5.29	0 46	6 57	4.84	5.16
1862	0 49	7 1	5.20	5.44	0 47	6 58	5.00	5.16	0 43	6 55	4.91	5.06
1863	0 47	6 57	5.19	5.14	0 48	7 0	5.25	5.05	0 53	7 4	4.99	5.24
1864	0 49	7 3	5.01	5.19	0 54	7 10	4.88	4.90	0 47	7 3	4.94	5.13
1865	0 52	7 6	4.91	5.06	0 50	7 5	4.83	5.06	0 50	7 6	4.91	5.01
Mean	0 47.6	6 58.7	5.082	5.266	0 46.5	6.57.8	5.113	5.3[illegible]0	0 45.3	6 57.2	5.091	5.360

Year.	OCTOBER.				NOVEMBER.				DECEMBER.			
	λ'_1	λ'_1	H'_1	H'_2	λ'_1	λ'_2	H'_1	H'_2	λ'_1	λ'_2	H'_1	H'_2
	h. m.	*h. m.*	*Ft.*	*Ft.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
1847	0 44	6 56	5.22	5.23	0 41	6 57	5.40	5.38	0 41	6 55	5.17	5.18
1848	0 44	6 57	5.30	5.72	0 40	6 54	5.07	5.39	0 37	6 52	5.23	5.48
1849	0 43	6 55	5.76	5.64	0 38	6 51	5.76	5.46	0 41	6 55	5.36	5.16
1850	0 41	6 52	5.21	5.25	0 40	6 53	5.16	5.18	0 41	6 54	5.30	5.21
1851	0 41	6 55	5.31	5.30	0 37	6 51	5.27	5.40	0 35	6 49	4.93	5.20
1852	0 40	6 53	5.31	5.85	0 42	6 55	5.34	5.68	0 32	6 47	5.30	5.45
1853	0 37	6 49	4.86	5.37	0 33	6 48	5.21	5.23	0 36	6 51	5.34	5.39
1854	0 38	6 43	5.09	5.70	0 38	6 51	5.19	5.52	0 40	6 54	5.03	5.26
1855	0 36	6 49	5.16	5.77	0 35	6 48	4.97	5.64	0 30	6 45	4.93	5.34
1856	0 41	6 53	5.17	5.80	0 45	6 47	5.17	5.71	0 41	6 42	5.21	5.25
1857	0 39	6 51	5.34	5.77	0 38	6 51	5.15	5.37	0 39	6 54	4.77	5.07
1858	0 38	6 50	5.02	5.62	0 37	6 49	5.38	5.37	0 38	6 36	4.90	5.25
1859	0 40	6 52	4.90	5.42	0 40	6 53	4.83	5.42	0 39	6 50	4.79	5.58
1860	0 44	6 56	4.83	5.18	0 43	6 57	5.08	5.63	0 40	6 54	4.93	5.37
1861	0 46	6 59	4.99	5.23	0 47	6 59	5.31	5.43	0 43	6 55	4.84	4.94
1862	0 43	6 53	5.00	5.28	0 44	6 52	4.97	5.16	0 42	6 53	4.72	5.00
1863	0 53	7 3	5.15	5.52	0 41	6 52	5.02	5.54	0 42	6 53	4.84	5.36
1864	0 48	7 2	5.05	5.26	0 41	6 57	5.14	5.12	0 36	6 49	5.10	5.10
1865	0 43	6 58	5.04	5.04	0 52	7 7	5.41	5.24	0 43	6 57	5.19	4.96
Mean	0 42.0	6 54.5	5.143	5.469	0 40.6	6 53.3	5.202	5.425	0 38.7	6 50.8	5.046	5.240

THE CONSTANT OR MEAN TIDE.

27. From the footings of Tables I and II we get the following table of average values of all the observations contained within certain limits of the argument $(\psi-\psi')$, and corresponding to the given average of the argument. These values, consequently, are independent of the effects depending upon any of the other arguments, and their inequalities depend only upon the argument $(\psi-\psi')$.

TABLE V.

UPPER TRANSITS.						LOWER TRANSITS.						COMBINED TRANSITS.					
Obs.	$(\psi-\psi')$	λ_1	λ_2	H_1	H_2	Obs.	$(\psi-\psi')$	λ_3	λ_4	H_3	H_4	Obs.	$(\psi-\psi')$	λ'_1	λ'_2	H'_1	H'_2
	h. m.	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*		*h. m.*	*h. m.*	*h. m.*	*Ft.*	*Ft.*
273	0 15.2	0 40.3	6 50.3	5.63	4.58	277	0 15.7	0 40.4	6 50.8	5.64	4.62	550	0 15.4	0 40.3	6 50.5	5.63	4.60
275	0 44.8	0 34.6	6 44.5	5.51	4.71	275	0 45.2	0 35.1	6 45.9	5.59	4.69	550	0 45.0	0 34.8	6 45.2	5.55	4.70
264	1 14.4	0 30.9	6 41.4	5.46	4.73	263	1 14.7	0 30.6	6 42.3	5.46	4.84	527	1 14.6	0 30.8	6 41.8	5.46	4.78
285	1 44.4	0 26.4	6 37.4	5.31	4.95	274	1 44.8	0 27.1	6 37.9	5.36	4.94	559	1 44.6	0 26.7	6 37.6	5.33	4.95
283	2 15.6	0 23.9	6 34.4	5.20	5.19	279	2 14.6	0 22.8	6 34.0	5.14	5.17	562	2 15.1	0 23.4	6 34.2	5.17	5.18
269	2 44.6	0 20.7	6 32.4	5.05	5.32	268	2 44.6	0 21.3	6 33.4	5.09	5.33	537	2 44.6	0 21.0	6 32.9	5.07	5.32
280	3 15.8	0 19.8	6 31.8	4.92	5.55	285	3 14.5	0 19.4	6. 31.4	4.89	5.55	565	3 15.2	0 19.6	6 31.6	4.90	5.55
280	3 44.2	0 20.3	6 32.6	4.74	5.71	281	3 45.6	0 20.3	6 33.9	4.74	5.73	561	3 44.9	0 20.3	6 33.2	4.74	5.72
283	4 15.9	0 23.2	6 36.7	4.58	5.88	283	4 14.8	0 22.6	6 36.3	4.58	5.81	566	4 15.3	0 22.9	6 36.5	4.58	5.85
283	4 44.8	0 26.4	6 40.0	4.50	5.91	272	4 45.8	0 27.6	6 42.2	4.48	5.99	555	4 45.3	0 27.0	6 41.1	4.49	5.95
283	5 15.6	0 32.3	6 47.8	4.48	5.97	273	5 14.3	0 32.0	6 47.2	4.48	5.99	556	5 15.0	0 32.1	6 47.5	4.48	5.98
288	5 44.3	0 37.9	6 52.8	4.46	6.06	271	5 45.0	0 39.1	6 53.8	4.39	6.00	559	5 44.7	0 38.5	6 53.3	4.43	6.03
276	6 15.5	0 45.9	7 0.9	4.46	6.03	291	6 14.0	0 44.6	7 0.2	4.47	5.96	567	6 14.7	0 45.3	7 0.6	4.46	6.00
283	6 44.2	0 50.8	7 6.3	4.54	5.79	289	6 46.0	0 52.6	7 7.0	4.51	5.85	572	6 45.1	0 51.7	7 6.6	4.52	5.82
271	7 15.4	0 58.4	7 13.1	4.62	5.72	278	7 14.8	0 56.4	7 11.6	4.64	5.55	549	7 15.1	0 57.4	7 12.4	4.63	5.63
284	7 44.2	1 1.0	7 16.1	4.84	5.49	273	7 45.2	1 2.5	7 17.0	4.81	5.52	557	7 44.7	1 1.7	7 16.5	4.82	5.50
281	8 15.8	1 5.9	7 18.6	4.87	5.33	287	8 14.6	1 4.6	7 17.5	4.95	5.31	568	8 15.2	1 5.3	7 18.0	4.91	5.32
280	8 44.5	1 4.8	7 17.1	5.22	5.06	275	8 45.9	1 5.4	7 18.2	5.06	5.10	555	8 45.2	1 5.1	7 17.6	5.14	5.08
282	9 15.2	1 4.8	7 16.2	5.21	4.94	272	9 15.5	1 3.6	7 16.5	5.34	4.90	554	9 15.3	1 4.2	7 16.3	5.27	4.92
276	9 44.9	1 1.5	7 12.9	5.49	4.79	268	9 45.4	1 2.6	7 13.8	5.42	4.88	544	9 45.1	1 2.0	7 13.3	5.46	4.83
275	10 15.0	0 59.1	7 9.8	5.50	4.80	276	10 14.7	0 57.4	7 9.7	5.61	4.67	551	10 14.9	0 58.2	7 9.8	5.55	4.73
280	10 44.3	0 53.7	7 4.3	5.72	4.58	264	10 45.5	0 53.8	7 4.8	5.59	4.64	544	10 44.9	0 53.7	7 4.5	5.66	4.61
278	11 15.3	0 49.0	6 59.7	5.68	4.50	280	11 15.1	0 47.2	6 59.8	5.68	4.55	558	11 15.2	0 48.1	6 59.7	5.68	4.53
268	11 45.1	0 45.4	6 55.3	5.67	4.58	269	11 45.7	0 44.2	6 54.8	5.68	4.50	537	11 44.7	0 44.8	6 55.0	5.67	4.54
	Means..	0 42.37	6 54.67	5.070	5.257		Means..	0 42.22	6 55.00	5.062	5.257		Means..	0 42.29	6 54.84	5.066	5.257

In the footings of this table the inequalities having the argument $(\psi-\psi')$ are also eliminated, and we have results belonging to the mean tide. Since the diurnal tide depends upon φ, it is also eliminated, and we have left only the constant part of the other oscillations.

With the preceding mean values of λ'_1 and λ'_2 we get from (44), supplying the omitted constant of 2 days,

$$(56) \qquad B_0 = \tfrac{1}{2}(2^d\ 0^h\ 42^m.29 + 2^d\ 6^h\ 54^m.84 - 6^h\ 12^m.62) = 2^d\ 0^h\ 42^m.25$$

which is *the mean establishment of the port* belonging to the assumed transit. In order to reduce this to the transit immediately preceding high water, we must add the constant part of k (34), putting $n=3$, and we thus get

$$B_0 = 2^d\ 0^h\ 42^m.25 - 1^d\ 13^h\ 13^m.14 = 0^d\ 11^h\ 26^m.53$$

From the first of (37) we get, since $L_2 = B_0$ in this case,

$$(57) \qquad \begin{cases} q_1 = 0\ 42.37 - 0\ 42.25 = 0^m.12 \\ q_2 = 6\ 54.67 - 0\ 42.25 = \tfrac{1}{2}\pi - 0^m.20 \\ q_3 = 0\ 42.22 - 0\ 42.25 = -0^m.03 \\ q_4 = 6\ 55.00 - 0\ 42.25 = \tfrac{1}{2}\pi + 0^m.13 \end{cases} \quad (\text{h. m.} \quad \text{h. m.})$$

in which $\pi = 12$ lunar hours or $12^h\ 25^m.24$ in solar time. Hence *all the intervals between high and low and low and high waters in the mean tide are almost exactly one-fourth of a lunar day.*

From the last two of (42) we get

(58) $$\begin{cases} K_3 \cos 3\,\Delta' = \tfrac{1}{2}(5.070 - 5.062) = .004 \\ K_3 \sin 3\,\Delta' = \tfrac{1}{2}(5.257 - 5.257) = .000 \end{cases}$$

Hence $\Delta'=0$ and $K_3=.004$ ft. This is the value of the constant or mean tertio-diurnal tide, and may be regarded as falling within the limits of the errors of observation, and consequently insensible.

With the preceding values of K_3, which is the constant and principal part of A_3, and q_n, the terms in the first of (42) are entirely insensible. We therefore obtain from (45), with the preceding mean values of H'_1 and H'_2,

(59) $$H'_0 = \tfrac{1}{2}(25.066 + 15.257) = 20.161 \text{ ft.}$$

for the mean height of the sea above the assumed zero of the tide-gauge. This, however, is not necessarily the same as the mean level obtained from observations made frequently at all times during the day, unless the tides follow the law of sines and cosines, or at least the parts above and below the mean level are symmetrical. It is simply the mean of the heights of high and low waters.

We obtain from the second of (42), since the terms depending upon K_3 are insensible,

(60) $$K_2 = \tfrac{1}{2}(25.066 - 15.257) = 4.904 \text{ ft.}$$

for *the coefficient of the average or mean semi-diurnal tide.* Consequently the mean range is 9.808 ft.

With the preceding values of K_3, Δ_1', and K_2, (43) gives

$$q_1 = 0, \qquad q_2 = \tfrac{1}{2}\pi, \qquad q_3 = -0^{m}.13, \qquad q_4 = \tfrac{1}{2}\pi + 0^{m}.13$$

These values from the formulæ depending upon the heights are almost exactly the same as those obtained above from the observed times alone, (57).

THE SEMI-MONTHLY INEQUALITY.

28. If from each of the values of λ'_1, in the preceding table, we subtract $B_0 + q_1 = 0^{h}\ 42^{m}.29$, omitting the constant two days, and from each value of λ_2' we subtract $B_0 + q_2 - k = 6^{h}\ 54^{m}.84 + 0^{m}.4 \cos \eta_1$, and call the differences δL, (46) and (47) will give, with these values of δL and the corresponding values of the argument $\eta_1 = 2(\psi - \psi')$, forty-eight equations of condition of the form

$$\delta L = M_1 \sin \eta_1 + N_1 \cos \eta_1 + M_{10} \sin \eta_{10} + N_{10} \cos \eta_{10}$$

the angles η_1 and η_{10} being the only ones included in the same argument. The tabular values of η_1 must be reduced to the time of low water for the twenty-four conditions obtained, from the low waters, by adding the mean change of η_1 from high to low water, which in this case is $25^{m}.24 + 0^{m}.4 \cos \eta_1$, the small term $0^{m}.4 \cos \eta_1$ being an inequality in the moon's motion depending upon the argument of variation, which is the same as η_1 or $2(\psi - \psi')$. With these forty-eight equations we obtain, by the method of least squares,

$$M_1 = -22^{m}.25, \quad N_1 = 0^{m}.01, \quad M_{10} = 1^{m}.96, \quad N_{10} = -0^{m}.37$$

These values satisfy the forty-eight conditions, with an average residual of $0^{m}.35$ and a maximum residual of $1^{m}.3$. The residuals do not indicate any sensible term depending upon $3\eta_1$. With these values (48) gives

(60) $$\begin{cases} B_1 = -22^{m}.5, & \epsilon_1 = 0^{\circ} \\ B_{10} = 2^{m}.0, & \epsilon_{10} = 5^{\circ} \end{cases}$$

These comprise the constants belonging to two of the terms of the expression of L_2, (26).

From (28) we get

(61) $$\tau_1 = B_0 = 2^{d}.03$$

for *the age of the tide from the times.*

If, from each value of H'_1 in the preceding table, we subtract $H'_0 + K_2 = 25.066$ ft., (59) and (60), and also from each value of H'_2 we subtract $H'_0 - K_2 = 15.257$ ft., and call the residuals δH, (50) gives forty-eight equations of the form

$$\delta H = K_0 (M_1 \cos \eta_1 + N_1 \sin \eta_1) \pm K_2 (M_1 \cos \eta_1 + N_1 \sin \eta_1 + M_{10} \cos \eta_{10} + N_{10} \sin \eta_{10})$$

in which the minus sign belongs to the conditions obtained from low waters, and the tabular values of η_1 for these conditions must be reduced to the time of low water as above. With these conditions we obtain, by the method of least squares,

(62) $$\begin{cases} K_0 M_1 = -0.059 \text{ ft.}, & K_2 M_1 = +0.670 \text{ ft.}, & K_2 M_{10} = -0.023 \text{ ft.} \\ K_0 N_1 = 0.000 \text{ ft.}, & K_2 N_1 = -0.124 \text{ ft.}, & K_2 N_{10} = +0.001 \text{ ft.} \end{cases}$$

These values satisfy the conditions with an average residual of .025 foot and a maximum of .06 foot. The residuals do not indicate any sensible term depending upon the angle $3\eta_1$ in the oscillations of the third kind, as may be also inferred from theory.

With the preceding values (49) gives

$$(63)\quad \begin{cases} K_0 R_1 = -0.059 \text{ ft.}, & R_{(0,1)} = 0.504, & \alpha_{(0,1)} = 0 \\ K_2 R_1 = +0.681 \text{ ft.}, & R_{(2,1)} = 0.1388, & \alpha_{(2,1)} = -10^\circ 29' \\ K_2 R_{10} = -0.023 \text{ ft.}, & R_{(2,10)} = -0.0047, & \alpha_{(2,10)} = -\ 2^\circ \end{cases}$$

These are the constants of two of the terms in the expression of A_2 and of one of the terms of A_0, (24).

In obtaining the values of R from K R the values of K_0 and K_2, (31) and (60), have been used. It will be remembered that R expresses the ratio of the inequality to the mean tide in each kind of oscillation. As the inequality $R_{(2,10)}$ of the second degree is scarcely sensible, it is not probable that there are sensible inequalities of the third degree.

From (28) we get, expressing the value of α_1 in terms of the radius,

$$(64)\qquad \tau_1 = 2^d.03 - \frac{.183}{.426} = 1^d.60$$

for *the age of the tide from the heights.*

INEQUALITY DEPENDING UPON THE MOON'S MEAN ANOMALY.

29. From the footings of Table III we get the following table of average values of all the observations contained within the limits of each of the twenty-four equal divisions of η_2, in which the middle of the division is taken as the value of η_2 belonging to the averages. In these footings the inequality depending upon $(\psi - \psi')$ is eliminated, and they consequently contain only the inequality depending upon η_2:

TABLE VI.

Obs.	η_2	λ'_1	λ'_2	H'_1	H'_2
	°	*h. m.*	*h. m.*	*Feet.*	*Feet.*
561	15	0 37.9	6 52.3	5.894	4.426
546	30	0 39.0	6 53.6	5.882	4.378
555	45	0 40.1	6 54.8	5.850	4.422
552	60	0 40.8	6 56.0	5.771	4.554
551	75	0 42.0	6 56.2	5.566	4.692
548	90	0 43.4	6 57.0	5.376	4.917
559	105	0 44.0	6 57.1	5.182	5.112
565	120	0 45.0	6 58.4	5.015	5.372
554	135	0 47.5	6 59.9	4.854	5.552
553	150	0 47.8	6 59.5	4.652	5.692
546	165	0 48.3	6 58.9	4.515	5.840
550	180	0 48.7	6 59.5	4.448	5.966
545	195	0 48.7	6 59.3	4.374	6.014
555	210	0 47.5	6 58.4	4.379	6.047
547	225	0 45.8	6 56.7	4.398	6.025
552	240	0 44.7	6 55.2	4.406	5.953
549	255	0 42.3	6 53.5	4.502	5.797
556	270	0 40.3	6 51.6	4.676	5.630
553	285	0 38.3	6 50.1	4.840	5.510
550	300	0 37.1	6 49.8	5.045	5.281
554	315	0 35.9	6 49.0	5.215	4.992
546	330	0 35.4	6 48.9	5.408	4.802
556	345	0 35.7	6 49.4	5.605	4.593
551	360	0 37.2	6 51.0	5.772	4.480
13,254	Means.	0 42.24	6 54.84	5.068	5.252

From the footings of this table the constants of the mean tide might also be obtained, as in the preceding case, but they would not differ sensibly, as may be seen by comparing the footings of the

two tables. If from each value of λ'_1 in the preceding table we subtract $B_0+q_1=0^h\ 42^m.24$, as obtained from the preceding table, and from each value of λ'_2 we take $B_0+q_2-k=6^h\ 54^m.84 +1^m.5\cos\eta_2$, putting δL for the residuals, (46) and (47) give forty-eight equations of the form

$$\delta L = M_2 \sin\eta_2 + N_2\cos\eta_2 + M_{11}\sin\eta_{11} + N_{11}\cos\eta_{11}$$

the angles η_2 and η_{11} being included in the same argument. In this case the value of k (34) includes the constant and the inequality depending upon η_2, the value of n being $\frac{1}{2}$ as before. In this case, to obtain the values of η_2 belonging to low water, we must add $3^\circ.3+0^\circ.5\cos\eta_2$ to the tabular values of η_2 given for high water. These conditions give, by the method of least squares,

$$M_2=3^m.3, \qquad N_2=-5^m.2, \qquad M_{11}=0^m.6, \qquad N_{11}=0^m.8$$

These values satisfy the conditions, with an average residual of $0^m.3$ and a maximum residual of $0^m.9$. With these values (48) gives

$$(65) \qquad \begin{cases} B_2 = 6^m.2, & \varepsilon_2 = \ \ 72^\circ.7 \\ B_{11}=1^m.0, & \varepsilon_{11} = -23^\circ \end{cases}$$

The preceding are the values of ε_2 and ε_{11} when η_2 is given for a time two lunar days after the transit C (§ 26). In order to reduce them to the case in which η_2 is given for the time of transit C, we must subtract the mean changes of η_2 and η_{11} in two lunar days, which is $27^\circ.1$ in the former and twice that, or $54^\circ.2$, in the latter. Hence we get, in this case,

$$(66) \qquad \varepsilon_2 = 45^\circ.6, \qquad \varepsilon_{11} = -77^\circ$$

The preceding are the constants belonging to two more of the terms in the expression of $\bar{L_2}$ (26).

If, now, as in the preceding case, we subtract H'_0+K_2 from each value of H'_1 in the preceding table, and H'_0-K_2 from each value of H'_2, with these forty eight residuals and the corresponding values of η_2, (50) gives forty-eight equations of the form

$$\delta H = K_0(M_2\cos\eta_2+N_1\sin\eta_2) \pm K_2(M_2\cos\eta_2+N_2\sin\eta_2+M_{11}\cos\eta_{11}+N_{11}\sin\eta_{11})$$

in which the minus sign belongs to low waters. From these forty-eight conditions we obtain

$$\begin{array}{lll} K_0\,M_2=-0.033 \text{ ft.}, & K_2\,M_2=+0.771 \text{ ft.}, & K_2\,M_{11}=+0.051 \text{ ft.} \\ K_0\,N_2=-0.014 \text{ ft.}, & K_2\,N_2=+0.204 \text{ ft.}, & K_2\,N_{11}=+0.014 \text{ ft.} \end{array}$$

With these values (49) gives

$$(67) \qquad \begin{cases} K_0\,R_2 = -0.036 \text{ ft.}, & R_{(0,2)} = 0.308, & \alpha_{(0,2)} = 38^\circ \\ K_2\,R_2 = +0.797 \text{ ft.}, & R_{(2,2)} = 0.1624, & \alpha_{(2,2)} = 29^\circ\ 51' \\ K_2\,R_{11} = +0.053 \text{ ft.}, & R_{(2,11)} = 0.0107, & \alpha_{(2,11)} = 45^\circ \end{cases}$$

These values satisfy the conditions, with an average residual of 0.020 ft. and a maximum residual of 0.049 ft. These are a part of the constants belonging to the terms in the expressions of A_0 and A_2 (24).

In order to reduce the preceding values of the angle of epoch to transit C, we must subtract $27^\circ.1$ from the first two and $54^\circ.2$ from the last one.

From (28) we get, with the reduced value of $\alpha_2=29^\circ.9-27^\circ.1$,

$$(68) \qquad \tau_2 = 2^d.03 + \frac{.049}{.229} = 2^d.24$$

INEQUALITY DEPENDING UPON THE MOON'S LONGITUDE.

30. By taking the average of all the values of λ and H of each transit in Tables I and II, and then taking the half sum and the half difference of the values of the two transits, we get the following table of average values belonging to the given longitude, from which the inequality depending upon $(\psi-\psi')$ is eliminated, and consequently they contain only the inequality depending upon the moon's longitude.

TABLE VII.

Obs.	Combined transits.					The differences of transits.			
	ϕ	λ'_1	λ'_2	H'_1	H'_2	$(\lambda_1-\lambda_3)$	$(\lambda_2-\lambda_4)$	(H_1-H_3)	(H_2-H_4)
	°	h. m.	h. m.	Ft.	Ft.	m.	m.	Ft.	Ft.
555	7.5	0 41.7	6 53.4	5.182	5.123	+2.0	−2.8	−0.486	+0.234
553	22.5	0 43.5	6 55.1	5.148	5.145	1.5	4.0	0.719	0.391
555	37.5	0 44.8	6 57.5	5.106	5.233	3.4	4.7	0.822	0.481
556	52.5	0 45.4	6 57.8	5.043	5.282	4.8	4.8	0.886	0.589
547	67.5	0 44.2	6 57.0	4.991	5.334	5.1	5.5	0.992	0.523
548	82.5	0 42.5	6 55.2	4.998	5.355	5.1	5.4	0.963	0.636
550	97.5	0 39.4	6 52.4	4.999	5.365	5.0	4.3	0.888	0.540
545	112.5	0 36.8	6 50.3	5.029	5.334	5.6	1.9	0.667	0.482
552	127.5	0 35.9	6 48.7	5.024	5.234	3.5	2.5	0.567	0.383
558	142.5	0 36.5	6 49.1	5.075	5.199	2.1	−2.2	−0.235	0.160
563	157.5	0 39.1	6 51.5	5.122	5.178	2.1	+0.1	+0.001	+0.042
553	172.5	0 43.0	6 54.4	5.131	5.168	+0.3	2.7	0.227	−0.068
566	187.5	0 45.2	6 56.9	5.131	5.157	−2.0	2.8	0.555	0.230
558	202.5	0 47.9	6 59.9	5.086	5.215	2.5	3.1	0.739	0.352
563	217.5	0 49.1	6 61.9	5.056	5.291	3.8	4.1	0.830	0.501
562	232.5	0 50.4	6 63.0	5.015	5.390	3.1	4.5	0.942	0.578
563	247.5	0 49.6	6 62.0	5.007	5.434	4.0	5.5	0.945	0.576
557	262.5	0 46.4	6 59.0	4.997	5.427	6.4	3.7	0.872	0.511
552	277.5	0 43.5	6 56.5	4.924	5.403	4.2	5.1	0.852	0.519
541	292.5	0 40.4	6 53.1	4.998	5.304	3.9	2.2	0.769	0.479
558	307.5	0 38.0	6 50.7	5.056	5.220	4.7	1.2	0.532	0.400
545	322.5	0 36.5	6 49.4	5.101	5.136	2.3	+0.1	0.328	0.206
569	337.5	0 37.4	6 50.3	5.168	5.097	1.6	−1.6	+0.042	−0.115
546	352.5	0 39.2	6 51.0	5.203	5.071	−1.5	−3.9	−0.291	+0.081
	Means.	0 42.35	6 54.83	5.067	5.255				

The inequalities in the values in the preceding table are affected by the tide produced by the small term in the moon's disturbing force depending upon the fourth power of the moon's distance. The expression of this tide, and also of the lunitidal interval, contains the angle φ, (33); and hence the expressions representing the inequalities in the preceding tabular values must contain such an angle. By proceeding in the same manner as in the preceding cases, we obtain from the preceding table forty-eight equations of the form,

$$\delta L = M'' \sin \varphi + N'' \cos \varphi + M_3 \sin \eta_3 + N_3 \cos \eta_3$$

These equations give

$$M'' = -2^m.1, \qquad N'' = -1^m.4, \qquad M_3 = 5^m.3, \qquad N_3 = 0^m.5$$

From these we get by (48),

(69) $$\begin{cases} B'' = 2^m.5, & \epsilon'' = -34^\circ \\ B_3 = 5^m.3, & \epsilon_3 = \quad 6^\circ \end{cases}$$

These constants satisfy the conditions, with an average residual of $0^m.4$ and a maximum of $1^m.4$.

From the values of H'_1 and H'_2 in the preceding table we obtain, as in the preceding cases, from (50) forty-eight equations of the form

$$\delta H = K_0(M_3 \cos \eta_3 + N_3 \sin \eta_3) \pm K_2(M_3 \cos \eta_3 + N_3 \sin \eta_3 + M'' \cos \varphi + N \sin \varphi$$

From these forty-eight conditions we obtain

$$\begin{array}{lll} K_0 M_3 = -.021 \text{ ft.}, & K_2 M_3 = +.107 \text{ ft.}, & M'' = +.030 \\ K_0 N_3 = +.005 \text{ ft.}, & K_2 N_3 = -.012 \text{ ft.}, & N'' = +.007 \end{array}$$

With these values (49) gives

(70) $$\begin{cases} K_0 R_3 = -.022 \text{ ft.}, & R_{(0,3)} = .190 & \alpha_{(0,3)} = -14^\circ \\ K_2 R_3 = \quad .109 \text{ ft.}, & R_{(2,3)} = .0225 & \alpha_{(2,3)} = -\ 6^\circ\ 30' \\ K_2 R'' = \quad .032 \text{ ft.}, & R'' \quad = .0065 & \alpha'' \quad = +13^\circ \end{cases}$$

Since the range of argument belonging to each group of observations is twice as great in this case as in the other cases, the coefficients of the inequalities, as obtained, are increased in the ratio of the sine to the arc of the half range of the groups of observations. An explanation of this small correction has been given in (§ 23). This very small correction is insensible in the other cases.

These constants satisfy the conditions, with an average residual of .016 ft. and a maximum of .057 ft. As the preceding inequalities of the first degree are so small, those of the second degree depending upon $2\eta_3$ must be very small and may be neglected.

From (28) we get

(71) $$\tau_3 = 2^d.03 - \frac{.113}{.460} = 1^d.78$$

Since the maximum of the small tide depending upon the fourth power of the moon's distance does not necessarily happen at the same time as that of the principal part of the semi-diurnal tide, the preceding value of $K_2 R''$ represents the height of that tide at the time of the high water of the resultant tide. Hence, putting

λ'' = the lunitidal interval of the small tide;

q'' = the time of the resultant high water after that of the mean semi-diurnal tide;

$K_2 a$ = the coefficient of the tide;

we have

$$1+R'' = \sqrt{1+a^2+2a\cos(L_2-\lambda'')} = 1+a\cos(L_2-\lambda'') \text{ nearly}$$

$$\tan q'' = \frac{a\sin(L_2-\lambda'')}{1+a\cos(L_2-\lambda'')} = a\sin(L_2-\lambda'') \text{ nearly}$$

From the preceding values of B'' and ε'' and of R'' and α'', we get

$$q'' = -2^m.5 \sin(\varphi+34^\circ)$$
$$R'' = .0065 \sin(\varphi+77^\circ)$$

These values of q'' and R'' cannot satisfy the preceding equations unless the angles are the same, whereas they differ 43°. But since the coefficient $K_2 R''$ of the small tide from which the value of α'' has been determined is only a small fraction of an inch, the discrepancy may be regarded as falling within the limits of the errors of observation. Assuming that the angles are equal, we then have, at the time of the maximum, $q'' = -2^m.5$, and $R'' = .0065$.

With these values the preceding equations give

$$.0065 = a\cos(L_2-\lambda'')$$
$$.0218 = -a\sin(L_2-\lambda'')$$

from which we get

(72) $$a = .023, \qquad L_2 - \lambda'' = -73^\circ$$

Hence $K_2 a = 0.115$ ft. is the coefficient of the tide, and 73° expressed in solar time, which is about $2\frac{1}{2}$ hours, is the time the high water of this small tide precedes the time of the high water of the principal tide. Hence *the high water of this tide occurs at* $11^h\ 26^m.5 - 2^h\ 30^m = 8^h\ 56^m.5$, *or about* 9 *o'clock in lunar time.* We also have in this case

(73) $$B''_0 = 2^d\ 0^h\ 42^m - 2^h\ 30^m = 1^d\ 22^h\ 12^m$$

for *the mean establishment.*

INEQUALITIES DEPENDING UPON THE SUN'S ANOMALY AND LONGITUDE.

31. From the means in the footings of Table IV we get the following table of average values, corresponding to the given values of v' and $2\varphi'$ belonging to the middle of each month:

TABLE VII.

Month.	′	$2\phi'$	λ'_1	λ'_2	H'_1	H'_2	$\frac{1}{2}(H'_1+H'_2)$	$\frac{1}{2}(H'_1-H'_2)$	$\delta H'_0$
	°	°	*h. m.*	*h. m.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*
January	15	230	0 37.9	6 51.0	4.933	5.190	20.061	4.861	—.089
February	45	290	0 40.7	6 52.2	4.841	5.060	19.950	4.890	—.200
March	75	351	0 40.8	6 53.4	4.962	5.058	20.010	4.952	—.140
April	105	51	0 41.4	6 54.1	5.000	5.177	20.092	4.915	—.058
May	135	110	0 43.4	6 55.5	5.075	5.253	20.164	4.911	+.014
June	165	168	0 46.0	6 57.8	5.090	5.225	20.157	4.932	+.007
July	194	226	0 47.6	6 58.7	5.082	5.266	20.174	4.908	+.024
August	223	284	0 46.5	6 57.8	5.113	5.300	20.206	4.906	+.056
September	252	344	0 45.3	6 57.2	5.091	5.360	20.225	4.865	+.075
October	282	44	0 42.0	6 54.5	5.143	5.469	20.306	4.837	+.156
November	313	106	0 40.6	6 53.3	5.202	5.425	20.313	4.888	+.163
December	344	168	0 38.7	6 50.8	5.046	5.240	20.143	4.903	—.007
Means			0 42.6	6 54.9	5.058	5.252	20.150	4.897	

From this table we obtain, as in preceding cases, twelve values of δL, which, together with the corresponding values of $v' = \eta_4$ and $2\varphi' = \eta_5$, in (46) and (47), give twelve equations of the form

$$\delta L = M_4 \sin \eta_4 + N_4 \cos \eta_4 + M_5 \sin \eta_5 + N_5 \cos \eta_5$$

From these we get, in the same manner as in preceding cases,

$$(74) \qquad \begin{cases} B_4 = -3^m.9, & \epsilon_4 = -73^\circ \\ B_5 = -0^m.6, & \epsilon_5 = -10^\circ \end{cases}$$

In the same way with the values of $\frac{1}{2}(H'_1 - H'_2)$, (50) gives twelve equations of the form

$$\delta H = K_2 (M_4 \cos \eta_4 + N_4 \sin \eta_4 + M_5 \cos \eta_5 + N_5 \sin \eta_5)$$

From these conditions we get

$$(75) \qquad \begin{cases} K_2 R_4 = 0.0378 \text{ ft.}, & R_4 = -.0077, & \alpha_4 = -65^\circ \\ K_2 R_5 = 0.0093 \text{ ft.}, & R_5 = -.0019, & \alpha_5 = -58^\circ \end{cases}$$

In the same manner we obtain from the values of $\frac{1}{2}(H'_1 + H'_2)$,

$$(76) \qquad \begin{cases} K_0 R_4 = 0.126 \text{ ft.}, & R_4 = 1.08, & \alpha_4 = 254^\circ \\ K_0 R_5 = 0.073 \text{ ft.}, & R_5 = 0.62, & \alpha_5 = 98^\circ \end{cases}$$

Since the arguments in the preceding table change so little from high to the following low water, $\frac{1}{2}(H'_1 + H'_2)$ may be taken as a normal value of the mean height, and $\frac{1}{2}(H'_1 - H'_2)$ as the coefficient or semi-range of the tide.

In the preceding table $\delta H'_0$ is the difference between any value of H'_0 and its mean value, and the column expresses the annual variation of mean level.

INEQUALITY DEPENDING UPON THE MOON'S NODE.

32. By summing the values of λ'_1, λ'_2, H'_1, and H'_2, and taking the averages so as to eliminate the annual inequalities, we obtain the following table of averages for each year, in which the value of ω belonging to the middle of the year is given:

TABLE VIII.

Year.	ω	λ'_1	λ'_2	H'_1	H'_2	$\frac{1}{2}(H'_1+H'_2)$	$\frac{1}{2}(H'_1-H'_2)$
	°	h. m.	h. m.	Feet.	Feet.	Feet.	Feet.
1848	156	0 42.5	6 55.7	5.21	5.24	20.22	4.99
1849	137	0 44.3	6 56.4	5.27	5.17	20.22	5.05
1850	118	0 42.4	6 54.6	5.21	5.17	20.19	5.02
1851	98	0 42.1	6 54.3	5.16	5.19	20.18	4.99
1852	79	0 41.8	6 54.8	5.15	5.30	20.22	4.93
1853	59	0 39.2	6 52.0	5.13	5.32	20.22	4.90
1854	40	0 40.0	6 53.1	4.98	5.28	20.11	4.83
1855	21	0 39.5	6 53.0	5.05	5.37	20.21	4.84
1856	2	0 40.2	6 51.0	5.04	5.49	20.21	4.78
1857	342	0 41.6	6 53.4	4.99	5.42	20.21	4.79
1858	323	0 41.8	6 52.2	4.86	5.33	20.10	4.77
1859	304	0 39.4	6 52.2	4.90	5.24	20.07	4.83
1860	285	0 42.5	6 55.5	4.86	5.11	19.99	4.88
1861	266	0 44.0	6 56.4	4.99	5.22	20.10	4.89
1862	246	0 43.6	6 55.1	5.00	5.25	20.12	4.88
1863	227	0 45.2	6 55.5	4.97	5.26	20.12	4.85
1864	207	0 45.1	6 58.7	5.02	5.16	20.09	4.93
1865	187	0 46.7	6 58.9	5.14	5.11	20.12	5.01
Means	....	0 42.6	6 54.9	5.048	5.249	20.150	4.897

In this case, as in the preceding one, the argument changes so slowly that we can take $\frac{1}{2}$ $(H'_1 + H'_2)$ as the mean level, and $\frac{1}{2}$ $(H'_1 - H'_2)$ as the mean range, corresponding to the given value of ω.

From the preceding values of λ'_1 and λ'_2, we obtain, in the same way as heretofore, eighteen equations of the form

$$\delta L = M_6 \sin \eta_6 + N_6 \cos \eta_6$$

which gives by (48),

(77) $$B_6 = -2^m.5, \qquad \epsilon_6 = -50^\circ$$

From the last column we obtain eighteen equations of the form

$$\delta H = M_6 \cos \eta_6 + N_6 \sin \eta_6$$

which, by (49), gives

(78) $$K_2 R_6 = -0.112 \text{ ft.}, \qquad R_6 = -.0235, \qquad \alpha_6 = -11^\circ$$

INEQUALITIES DEPENDING UPON η_8 AND η_9.

33. If in Table III we combine all the values of λ'_1, λ'_2, H'_1, and H'_2, in which $\eta_1 + \eta_2 = 7^\circ.5$, and then all those in which $\eta_1 + \eta_2 = 22^\circ.5$, and so on; and likewise combine all those in which $\eta_1 - \eta_2 = 7^\circ.5$, and then all those in which $\eta_1 - \eta_2 = 22^\circ.5$, and so on, we get the following table of averages, corresponding to twenty-four values of the argument $\eta_1 + \eta_2 = \eta_8$, and also to twenty-four values of the argument $\eta_1 - \eta_2 = \eta_9$, in which the inequalities of all the other arguments are eliminated:

TABLE IX.

η_8	λ'_1	λ'_2	H'_1	H'_2	η_9	λ'_1	λ'_2	H'_1	H'_2
°	*h. m.*	*h. m.*	*Feet.*	*Feet.*	°	*h. m.*	*h. m.*	*Feet.*	*Feet.*
7.5	0 41.7	6 54.3	5.104	5.232	7.5	0 43.4	6 55.9	5.195	5.123
22.5	0 43.0	6 55.0	5.099	5.242	22.5	0 45.0	6 57.7	5.119	5.147
37.5	0 42.2	6 55.2	5.073	5.247	37.5	0 45.1	6 57.8	5.095	5.206
52.5	0 43.4	6 55.9	5.036	5.207	52.5	0 45.3	6 58.0	5.082	5.254
67.5	0 43.4	6 55.8	5.015	5.247	67.5	0 45.8	6 58.2	5.017	5.253
82.5	0 43.8	6 56.9	5.060	5.252	82.5	0 45.5	6 58.7	5.034	5.295
97.5	0 43.9	6 56.6	5.030	5.270	97.5	0 45.4	6 57.8	4.998	5.372
112.5	0 44.2	6 56.1	5.029	5.283	112.5	0 44.7	6 57.2	5.016	5.337
127.5	0 43.4	6 56.0	4.978	5.259	127.5	0 43.8	6 56.5	4.999	5.386
142.5	0 43.5	6 55.9	5.082	5.252	142.5	0 43.2	6 55.3	4.985	5.364
157.5	0 43.2	6 55.8	5.025	5.290	157.5	0 41.5	6 54.3	4.961	5.390
172.5	0 42.8	6 55.0	5.030	5.278	172.5	0 41.2	6 53.8	4.943	5.343
187.5	0 42.5	6 55.2	5.089	5.296	187.5	0 40.4	6 52.6	4.926	5.352
202.5	0 42.2	6 55.2	5.107	5.254	202.5	0 39.5	6 51.7	4.997	5.332
217.5	0 41.4	6 53.3	5.039	5.290	217.5	0 38.7	6 51.8	4.989	5.267
232.5	0 41.3	6 54.4	5.067	5.242	232.5	0 39.9	6 52.2	5.060	5.290
247.5	0 40.8	6 53.8	5.111	5.286	247.5	0 39.1	6 51.9	5.108	5.260
262.5	0 41.0	6 53.5	5.090	5.244	262.5	0 39.1	6 51.0	5.111	5.196
277.5	0 40.6	6 53.6	5.077	5.217	277.5	0 39.3	6 51.9	5.130	5.227
292.5	0 40.4	6 53.0	5.067	5.238	292.5	0 39.5	6 52.1	5.144	5.143
307.5	0 41.2	6 53.7	5.102	5.254	307.5	0 40.3	6 52.8	5.167	5.147
322.5	0 40.3	6 53.8	5.108	5.220	322.5	0 41.5	6 54.8	5.183	5.130
337.5	0 41.8	6 53.7	5.080	5.219	337.5	0 42.4	6 55.0	5.183	5.103
352.5	0 41.5	6 54.5	5.095	5.242	352.5	0 44.2	6 57.1	5.139	5.128
Means.	0 42.2	6 54.8	5.066	5.253	Means.	0 42.3	6 54.8	5.066	5.252

By the methods heretofore used, we obtain from these tabular results,

$$(79)\qquad \begin{cases} B_8 = 1^m.5, & \epsilon_8 = -25°, & K_2 R_8 = .0265 \text{ ft.}, & R_8 = .0054, & \alpha_8 = -45° \\ B_9 = 3^m.5, & \epsilon_9 = 24°, & K_2 R_9 = .1196 \text{ ft.}, & R_9 = .0240, & \alpha_9 = -21° \end{cases}$$

As has been stated, the values of η_2 in Table III belong to a time two lunar days after transit C, and hence the values of the argument η_8 are too great for the assumed transit C, by the change of η_2 in that time. For the same the values of argument η_9 are too small by that amount. The preceding values of the epochs have been reduced to transit C by subtracting 27°.1 in the former case, and adding the same amount in the latter case.

In the same manner the constants might be found for the terms depending upon the arguments $\eta_1+\eta_3$ and $\eta_1-\eta_3$, or $\eta_2+\eta_3$ and $\eta_2-\eta_3$, but the effects depending upon these arguments must be still smaller than those depending upon η_8 and η_9. This one case will serve to show that these effects are of very little importance.

DIURNAL TIDE.

34. It has been found that the terms depending upon A_3 are insensible, and hence with the twenty-four values of (H_1-H_3) and (H_2-H_4) in Table V, (52) gives twenty-four sets of equations for determining the twenty-four values of A_1 and Δ, belonging to the twenty-four values of φ. With these values of A_1 and the corresponding values of φ in the table, (53) then gives twenty-four equations of condition for determining, by the method of least squares, the values of M and N, with which we then obtain, by (54),

$$(80)\qquad K_1 = -0.58 \text{ ft.}, \qquad \alpha = -19°.7$$

for the coefficient of the diurnal tide and the value of the angle at the epoch.

From (28) we get, using the value of B_0 in (56),

$$(81)\qquad \tau = 2^d.03 - \frac{19°.7}{13°.18} = 0^d.504$$

for *the age of the diurnal tide*, 13°.18 being the daily motion of the moon in longitude.

The preceding twenty-four equations of conditions also give $\Delta = 31°$. Expressing this value in solar time, we get from (37) $L_2 - L_1 = 2^h\ 8^m$ for *the time the high water of the diurnal tide precedes that of the semi-diurnal tide.*

We consequently have

(82) $$B_0 = 2^d\ 0^h\ 42^m - 2^h\ 8^m = 1^d\ 22^h\ 34^m$$

for *the mean establishment of the diurnal tide* belonging to transit C.

It is evident from (41) that $\frac{1}{2}(\lambda_1 - \lambda_3) = q_1$ and $\frac{1}{2}(\lambda_2 - \lambda_4) = \frac{1}{2}\pi - q_2$. With the preceding values of K_1 and Δ_1 and K_2 (60), we get from (55), expressing arcs in time,

$$q_1 = +1^m.8 \sin(\varphi - a)$$
$$\tfrac{1}{2}\pi - q_2 = -3^m.0 \sin(\varphi - a)$$

in which a must have the value above. Hence these expressions should represent $\frac{1}{2}(\lambda_1 - \lambda_3)$ and $\frac{1}{2}(\lambda_2 - \lambda_4)$ in Table VII. The angle of the epoch is about right, but the coefficient of the former is nearly one minute too small and that of the latter a little too great. These slight discrepancies are, no doubt, caused by the existence of a small quarter-day tide, which has not been taken into account in (41), from which the preceding expressions have been deduced.

RECAPITULATION OF RESULTS.

35. For the general tidal expressions, (24) and (26), the following constants have been obtained, in which the values of the epochs are for transit C:

In oscillations of mean level, in which $s = 0$,

(83) $$\begin{cases} K_0 = -0.117 \text{ ft.}, & \\ R_1 = 0.504, & a_1 = 0 \\ R_2 = 0.308, & a_2 = -4 \\ R_3 = 0.190, & a_3 = -14 \\ R_4 = 1.08, & a_4 = +74 \\ R_5 = 0.62, & a_5 = -82 \end{cases}$$

In diurnal oscillations, in which $s = 1$, (32),

(84) $$K_1 = -0.58 \text{ ft.}, \qquad a_1 = -19°.7, \qquad B_0 = 1^d\ 22^h\ 34^m$$

In semi-diurnal oscillations, in which $s = 2$,

(85) $$\begin{cases} K_2 = 4.904 \text{ ft.}, & & B_0 = 2^d\ 0^h\ 42^m.25, & \\ R_1 = 0.1388, & a_1 = -10°.5, & B_1 = -22^m.6, & \varepsilon_1 = 0° \\ R_2 = 0.1624, & a_2 = +2°.7, & B_2 = 6^m.2, & \varepsilon_2 = 45°.6 \\ R_3 = 0.0225, & a_3 = -6°.5, & B_3 = 5^m.3, & \varepsilon_3 = 6° \\ R_4 = -0.0077, & a_4 = -65°, & B_4 = -3^m.9, & \varepsilon_4 = -73° \\ R_5 = -0.0019, & a_5 = -58°, & B_5 = -0^m.6, & \varepsilon_5 = -10° \\ R_6 = -0.0235, & a_6 = -11°, & B_6 = -2^m.5, & \varepsilon_6 = -50° \\ R_8 = 0.0054, & a_8 = -45°, & B_8 = 1^m.5, & \varepsilon_8 = -25° \\ R_9 = 0.0240, & a_9 = -21°, & B_9 = 3^m.5, & \varepsilon_9 = 24° \\ R_{10} = -0.0045, & a_{10} = -2°, & B_{10} = 2^m.0, & \varepsilon_{10} = 5° \\ R_{11} = 0.0107, & a_{11} = -9°, & B_{11} = 1^m.0, & \varepsilon_{11} = -77° \end{cases}$$

In tertio-diurnal oscillations, in which $s = 3$,

(86) $$K_3 = 0.004 \text{ ft.}$$

In the semi-diurnal tide, depending upon the fourth power of the moon's distance, (36),

(87) $$K'' = .0065, \qquad a'' = -77°, \qquad B_0 = 1^d\ 22^h\ 12^m$$

The absolute values of the coefficients of the tide are $K_s R_i$.

COMPARISONS WITH THE EQUILIBRIUM THEORY.

36. The values of the constants K_s in the equilibrium theory, and also in the dynamic theory in the oscillations of mean level, are given approximately in (31). By comparing these values with

the preceding, it is seen that K_1 and K_3 given by observation are less than the theoretical values, while that of K_2, the mean coefficient of the semi-diurnal tide, is nearly ten times greater. But in applying the equilibrium theory to the real case of nature, it has been usual to determine such constants as make the expressions best represent the observations instead of determining them from theory, and to depend upon the theory for the ratios of the inequalities to these constants. In the equilibrium theory, and also in the dynamic theory in the case of oscillations of mean level, we should have $R_n = P_n$. By comparing the preceding values of R_n with those of P_n (12) it is seen that while P_1 is wanting, $R_1 = .504$, and also that the value of R_2 is greater than that of P_2. The three values of $(R_n - P_n)\, K_o$ in these three cases give respectively -0.059 ft., -0.020 ft., and $+0.003$ ft. as the coefficients of inequalities in the mean level, belonging respectively to the arguments η_1, η_2, and η_3, for which there are no corresponding inequalities in the disturbing forces. A semi-monthly inequality of mean level, corresponding with the first of the preceding, and with the same sign, though frequently much greater, has usually been found in all tidal discussions.

The parts of the preceding inequalities without any corresponding disturbing forces are, no doubt, the effects of a quarter-day tide, which, with observations of high and low waters only, there are no means of separating from the semi-diurnal tide, and are not inequalities in the true mean level (§ 27); and the preceding inequalities are merely the inequalities in this tide, which varies with the semi-diurnal tide, and the effect of the constant part of this tide is contained in the value of H'_0. Upon this hypothesis, if we assume the existence of a quarter-day tide with a coefficient of about three inches, it would account for these inequalities within the limits of the errors of observation. We have already had indications of the existence of such a tide elsewhere (§ 34). The exact coefficient of such a tide can only be determined from observations made several times during the phase of the tide.

If we compare the values of R_4 (83) with that of P_4 in (12), we see that the latter is a very inconsiderable part of the former, and that the difference corresponds to an annual inequality of mean level with a range of about four inches, for which there is no corresponding disturbing force. The whole of this inequality is given for the middle of each month in the last column of Table VII. The maximum of this inequality occurs in October or November and the minimum in February. Such an inequality has been found at other ports. At Brest it is a little greater, with the maximum and minimum occurring a little later in the year. Dr. Bache found, from the discussion of the tides at Key West, an annual inequality with a range of about nine inches, and with the maximum in September and the minimum in February.

These results should not be regarded as being at variance with the general tidal theory, but merely as being the effects of some circumstances or causes not taken into account in the theory; and these effects are, no doubt, due to the annual changes in the currents of the ocean, produced by annual changes of temperature and of the winds. On account of the influence of the earth's rotation there cannot be an annual change in the velocity or position of the currents of the ocean without a corresponding change generally in the mean level of the ocean at any port. The preceding results are very interesting in connection with this subject. I endeavored to give a full explanation of these inequalities, a few years ago, in the *Proceedings of the American Academy of Arts and Sciences*, Vol. VII, p. 31.

By comparing the values of R_n (85) with those of P_n (18), it is seen that they differ very much, and consequently the equilibrium theory, applied to the Boston tides, gives very erroneous relations between the inequalities and the mean tide. While $R_1 = .1388$, we have $P_n = .4240$, and hence the equilibrium theory would make the semi-monthly inequality in heights more than three times greater than it is. In the same way it is seen that it makes the inequality depending upon the moon's parallax too small, while it makes that depending upon the moon's longitude, or declination, more than four times greater.

37. In the equilibrium theory the lunitidal interval is expressed by $\frac{2\,\beta_2}{i}$ (22), and the coefficients of the inequalities in the development by Q_n (23). The mean establishment, referred to the nearest transit, should be 0, which does differ much from observation at Boston. But if we compare the preceding value of B_1 with that of Q_1 (23), we see that the equilibrium theory gives nearly two and a half times that of observation for the coefficient of the semi-monthly inequality. For the observed

inequality also of $6^m.2$ depending upon η_2, the equilibrium theory gives none; and throughout the smaller coefficients there are large proportionate differences between the theory and observation.

38. The values of the angles of epoch in the equilibrium theory should be 0 when referred to the nearest transit. The values of α_1 and α_2 (83) both nearly vanish for the transit C occurring two lunar days earlier, and the value of α_3 would vanish for a transit nearly a day and a half earlier. This also applies to the dynamic theory. But it has been shown that these inequalities, except a small part of the second and third, do not depend upon any corresponding term in the disturbing force, but are probably the effects of quarter-day tides, resulting from the circumstance of a shallow sea, and depending upon the magnitude and epoch of the semi-diurnal tide. Hence the values of the angles of the epoch should correspond somewhat with those of the semi-diurnal tide in (85), which they do as nearly as could be expected, since it is impossible to determine the values of the angles accurately for so small inequalities. The preceding comparisons are sufficient to show how inadequate the equilibrium theory is to represent the observed inequalities of the Boston tides.

DETERMINATION OF THE GENERAL CONSTANTS.

39. It is now proposed to determine the constants in the general tidal expressions (25) and (27) These being known, these expressions then give the special constants belonging to each inequality. Among the constants to be determined is the correction of the moon's mass, $\delta\mu$, contained in the expressions of P_i, U_i, and Q_i, (18) and (27). The diurnal tide in the port of Boston being very small, only the coefficient of the principal inequality has been determined from observation, and consequently we have no means of forming conditions enough to determine these general constants belonging to this tide; and they are of no consequence, since the effects depending upon them in this case must be insensible. We can, therefore, only determine these constants for the semi-diurnal tide.

With the values of R_i (85) belonging to the first three inequalities, and the corresponding values of P_i and U_i, the first of (25) gives the following conditions:

$$(88)\qquad \begin{cases} .1388\,(1+F)=.4305-24.0\,\delta\mu-(.1742-13.2\,\delta\mu)E \\ .1624\,(1+F)=.1521+\ \ 3.6\,\delta\mu+\ .0500\ E \\ .0225\,(1+F)=.0985+\ \ 1.0\,\delta\mu-(.0477-0.5\,\delta\mu)E \end{cases}$$

Also with the values of $M_i=B_i\cos(\varepsilon_i-\alpha_i)$, belonging to the same inequalities, the third of (27) gives

$$(89)\qquad \begin{cases} -22^m.2=-52^m.5+4034\,\delta\mu+(21^m.75-1210\,\delta\mu)E-0^m.6 \\ \ \ 4^m.3=\qquad\qquad\qquad\qquad +(\ 4^m.10+\ 187\,\delta\mu)E-0.3 \\ \ \ 5^m.3=-\ \ 2^m.2+\ 148\,\delta\mu+(\ 5^m.16+\ \ 50\,\delta\mu)E-0.1 \end{cases}$$

By giving proper relative weights to these two sets of conditions from the heights and from the times, we can combine them in the determination of the constants, and thus obtain the most probable values which all the conditions give, and get some idea of their probable errors. These six are the only conditions which can be formed having much weight in the determination.

A solution of the first three conditions, (88), depending upon the heights of the tides, gives

$$(90)\qquad \delta\mu=-.000283,\ E=1.408,\ \text{and}\ F=.365$$

With this value of $\delta\mu$, (9) gives for the moon's mass,

$$\mu=.013-.000283=.012717=\frac{1}{78.64}$$

In order to determine the most probable value of the three preceding constants belonging to all of the preceding conditions, we shall multiply the first three by 300 and then substitute the preceding values plus the correction belonging to the new conditions. We thus get

$$\begin{array}{rcl|l} & & & \text{Residuals.}\ \ \text{m.} \\ -1650\,\Delta\mu-51.60\,\Delta E-41.64\,\Delta F & = & 0 & \ \ \ .00 \\ +1080\,\Delta\mu+15.00\,\Delta E-48.72\,\Delta F & = & 0 & +\ .02 \\ +\ 510\,\Delta\mu-14.31\,\Delta E-\ 6.75\,\Delta F & = & 0 & -\ .12 \\ +2333\,\Delta\mu+21.75\,\Delta E & = & 0.3 & +0.17 \\ +\ 262\,\Delta\mu+\ 4.10\,\Delta E & = & -1.2 & -1.03 \\ +\ 218\,\Delta\mu+\ 5.16\,\Delta E & = & 0.3 & +0.29 \end{array}$$

The solution of these by the method of least squares gives

$$\Delta\mu = .000120 \pm .00021, \quad \Delta E = -.005, \text{ and } \Delta F = .001$$

with the residuals given above. The first three residuals divided by 300 and multiplied by 60 inches, the coefficient nearly of the mean tide, give the real residuals in inches, the greatest of which is only .024 inch. The last three are the residuals belonging to the times in minutes. The multiplication of the conditions from the heights by 300 makes the relation between the two kinds of residuals in general about the same as that of the probable errors of the two kinds of observations.

With the preceding corrections we get for the constants belonging to the new set of conditions,

$$(91) \qquad \begin{cases} \mu = 0.012717 + (.000120 \pm .00021) = \dfrac{1}{77.9 \pm 1.3} \\ E = 1.408 - .005 = 1.403 \\ F = 0.365 + .001 = 0.366 \end{cases}$$

The preceding residuals show the accuracy with which theory represents the three principal inequalities of the heights and the times. This may be somewhat accidental in this case, and it remains yet to be determined by an application of the theory to other ports, whether it will represent the inequalities of the times with such accuracy that conditions deduced from them should have any weight in the determination of the moon's mass. The preceding probable error, obtained from so few conditions, cannot be relied upon as showing with much certainty the real probable error of such determinations in general.

The preceding value of E for the port of Boston is four or five times as great as it is in most European ports. This extraordinary value, in the first of (25), diminishes the values of R_1 and R_3, and consequently the coefficients of the first and third inequalities, to less than one-third of what they would be by the equilibrium theory, but increases the second inequality, since U_2 is negative, and makes it greater than what the equilibrium theory would require, and even greater than the semi-monthly inequality. In like manner this large value of E, in the third of (27), diminishes the coefficient, and consequently the range, of the first or semi-monthly inequality in the lunitidal intervals, so that at Boston it is only about half as much as in European ports generally, and less than half of what the equilibrium theory requires.

The value of F above, in the first of (25), tends to decrease all the inequalities, and in the first and third is in the same direction with the effect of the term depending upon E; but in the second inequality the effects of the two terms are in contrary directions, but that depending upon E is the greater, and consequently the second inequality is greater than the conditions of a static equilibrium would require. Whether the effect of the term depending upon F is due to friction, as we have supposed, (§ 14), or to some other cause, it is evident that the preceding conditions cannot be satisfied without such a term.

40. In the equilibrium theory both E and F vanish, and in this case the first of either (88) or (89) furnishes a condition for determining the correction of the moon's mass, the former from the heights, and the latter from the times. It is well known that this theory, even in European ports, where the deviation from the relations of the equilibrium theory is comparatively small, gives very unreliable determinations of the moon's mass; but in the port of Boston it would give a mass more than double the true mass, as may be seen from a mere inspection of the conditions in this case. The preceding conditions are based upon the hypothesis of a small correction only, and consequently fail in this case, except that they indicate that it must be very large.

In the equilibrium theory only one condition is necessary for determining the moon's mass, which can be based upon the first and principal inequality. Where the term depending upon E has a sensible effect, a second condition is necessary, which may depend either upon the second or third inequalities. Laplace, in his last tidal investigations in the fifth volume of the *Mécanique Celeste*, used the first and third inequalities. Airy, in the discussion of the tides of Ireland, formed conditions from the first and second inequalities, but obtained a mass very much too great. When the terms depending upon F in the preceding conditions have a sensible value, it is readily seen that these two sets of conditions must give very different results, and that the conditions based upon the first and third inequalities must be much better than those based upon the first and

second, since the effects of the two terms depending upon E and F are in the same direction in the former conditions, and somewhat in the same proportion, while in the latter they are in contrary directions.

When the terms depending upon F are sensible, there are three quantities to be determined, and consequently three conditions are necessary, and two of these conditions, if the heights alone are used, have to depend upon the small parallactic and declination inequalities belonging to the moon. Where circumstances are such as to make the terms depending upon E and F small, as at Brest, the conditions are sufficient to determine them with adequate precision to get a pretty accurate determination of the moon's mass; but in the case of the Boston tides these become terms of a first order, which, in a great measure, destroy the first and third inequalities, upon which the determination mainly depends, so that the problem becomes nearly indeterminate, and the conditions are not sufficient to give a reliable determination of the moon's mass; for the small semi-monthly inequality observed is not wholly due to the solar tide, which is a kind of base in the determination, but a considerable part of it belongs to that term in the moon's parallax depending upon the argument of variation, and which gives no advantage in forming the conditions.

By the preceding method, in which conditions from the times are taken in, which our tidal expressions enable us to do, the great weight of the determination which rests upon the small declination inequality, which, in the case of the Boston tides, is reduced to 1.3 inches, is thrown upon the principal semi-monthly inequality of the lunitidal intervals, which is $22^{m}.6$, and which can be observed with much greater accuracy, proportionately, than 1.3 inches in the heights. The two other conditions from the times also give some little additional weight to the determination. The accuracy of the determination, of course, depends very much upon the accuracy of the tidal expressions, and the probable error may be greater than that which we have obtained from so few conditions, but still I think the determination is entitled to considerable weight. But it is very evident, from what has been stated, that the Boston tides are not at all favorable for an accurate determination of the moon's mass, and that the same magnitude of errors in theory or observation makes the probable error of the determination several times greater than in the case of tides, as those of Brest, in which the magnitudes of the inequalities differ but little from what the equilibrium theory would give.

COMPARISONS WITH THE DYNAMIC THEORY.

41. In the oscillations of the first kind, which are oscillations of long period, the terms depending upon E, F, and G vanish, and the dynamic corresponds with the equilibrium theory, with which comparisons have already been made. The diurnal tides of Boston being very small, only the constants of the principal term depending upon the moon's longitude have been determined from the observations; and, as it requires at least two conditions to determine the constants in the expressions (25) and (27) for each kind of oscillation, we have no means of making any comparisons of results obtained from the observations with those given by these expressions. The constants E and F having been determined for the semi-diurnal tide, (90), the first of (25) should give the value of R_i for each of the inequalities of which we know the value of P_i; and K_2 being known we then have $K_2 R_i$, the coefficients of the inequalities. The third of (27) should likewise give $M_i = B_i \cos(E_i - \alpha_i)$. Three of the values of R_i and M_i, obtained from observation, belonging to the three principal inequalities, have been used in the six conditions by which the constants have been determined, and, from the smallness of the residuals, it is seen that, so far as these three inequalities are concerned, the observations are well represented by theory.

The values of τ, obtained from (28), with the angles of epoch α_i, should all be equal, according to theory, taken in terms of a first order only. But, we see from (64), (68), and (71), these values differ considerably at Boston. Neglected terms of a second order, which are very large at Boston, are, no doubt, sensible in this case. It is difficult, also, to obtain the angles of epoch of small inequalities of long period with much accuracy from the observations, but in this case the differences seem to be rather great to be attributed to errors of observation. The other inequalities are too small to give a reliable value of τ.

The values of the angles of epoch E_i, in the inequalities of the intervals, should be given by the second of (27), but no value for F' in the expression of N_i can be obtained which will represent them

accurately, and there are evidently some neglected sensible terms which affect this expression. But these angles depend upon very small quantities, since the coefficients are mostly very small, and consequently a very small effect throws them very much out.

In the fourth and fifth inequalities of the heights depending upon the sun's parallax and declination, the observed inequalities are both quite small, less than a half-inch, as theory requires; but here also there are some disturbing influences not represented in the theory, for the coefficients have the contrary sign, and the angles of the epoch, which in this case should be sensibly 0, since $D_t\eta_4$ and $D_t\eta_5$ are insensible in the expression (25), are quite large.

These disturbing effects belong mostly to the fourth inequality having an annual period, and are, no doubt, due in part to the varying effects of friction, caused by annual variations in the velocities and positions of ocean-currents, as the Gulf Stream; for such variations, depending upon the changes of the seasons and of temperature, must have an annual period. We have seen that in the oscillations of mean level also there is a very considerable annual inequality not indicated by theory, which we have supposed to be due to influences of the same kind, (§ 36). Now, any amount of change of mean level, from whatever cause, must also produce a slight corresponding effect upon the range of the tidal oscillations and also upon the time, which, in very shallow seas and harbors, may be quite sensible to observation. According to theory, the value of B_4 (85) should be 0; hence we have an annual inequality in the times with a coefficient of four minutes, which must be due to the same causes, as the angle of the epoch, corresponding very nearly with that in the inequality of the heights, seems also to indicate. These seeming deviations from theory are merely the effects of slight disturbing influences not taken into account in the theory, and should be regarded as very important in the investigation of the subject of tidal friction in connection with ocean-currents varying with the seasons.

We come now to the sixth inequality, depending upon the moon's node. In this case U_6 in the first of (25) being sensibly nothing, we should have $(1+F)\,R_i = P_i$, or substituting the value of P_6 (18), and the observed value of R_6 (85), we should have $.0235(1+F)=.0375$. This gives the value of F, as deduced from this small inequality alone, equal to nearly .6, which is larger than the value before obtained, from conditions from all the principal inequalities. With the value .401 before obtained, the preceding equation gives $R_6=-.0268$, and consequently the tidal coefficient given by the tidal expression is too great by $-(.0268-.0235)\times 60$ inches, or about one-fifth of an inch. If $F=0$, then the observed value of R_6 should be equal to P_6, the value of R_6 belonging to static equilibrium. But the difference is nearly an inch, which is entirely too great to be attributed to errors of observation; and hence the comparison of observation with theory, in the case of this small inequality alone, shows that F must have a sensible value, and that all the terms depending upon it which have never before been taken into account in any tidal theory, must, in the port of Boston at least, have very sensible values.

The value of $B_6=-2^m.5$ (85) in this case must depend upon F' in the expression of N_6 (27), since all the other terms in the expression vanish in this case. This indicates that F should have a positive value, as is also required in the first inequality. The effect of terms depending upon F^1 then is to cause the lunitidal intervals of larger tides to be a little greater than those of smaller ones, and consequently neglecting terms of a third order, to introduce small inequalities into the intervals proportional to the inequalities in the heights.

All the remaining coefficients of the inequalities are quite small and unimportant, and, in the comparison of them with the tidal expressions as here given, there is not a very nice agreement, some of the residuals being nearly an inch. But the correct tidal expressions for these inequalities of a second order depend upon so much development that the more simple expressions, as given in the preceding pages, in which many terms are necessarily neglected, cannot be expected to give accurately these small inequalities in this case on account of the neglected terms, which, although in most cases insensible in the Boston tides, on account of the large value of E, must be quite sensible. The coefficients of the remaining inequalities in the times, as given by observation, agree very well with those given by the tidal expressions, none of the residuals being more than $0^m.5$.

42. All solutions of the tidal problem, extended to the cases of different and varying motions of the disturbing bodies in right ascension assume, as Laplace substantially expresses it, that if the tidal coefficient changes with any change of the motion of the disturbing body in right ascen-

sion, the corresponding changes when small may be regarded as exactly proportional. This is equivalent to supposing that in the development of the tidal coefficient K, which is a function of i, by Taylor's theorem, only the first term depending upon δi, of which the coefficient is D_i K or E, is sensible, and that all the others, depending upon the square and higher powers of δi, may be neglected. This is, no doubt, true of Brest, and of most European ports, but we are hardly safe in assuming that it is strictly true in the case of the Boston tides; for we have seen that the effect of the first term, that depending upon E, is a quantity of the first order of the inequalities, and in the case of the semi-monthly inequality amounts to more than a fourth part of the whole tidal coefficient, and hence it can hardly be supposed that the second term depending upon $(\delta i)^2$ is entirely insensible. If such terms are sensible they must cause slight deviations of observation from theory, and also affect the determination of the moon's mass. The smallness of the residuals, however, do not indicate that the effect of such terms, if at all sensible, can be of much consequence.

PREDICTION FORMULÆ AND TABLES.

43. With the preceding tidal expressions the heights and times of the tides may be computed for any given time; but although these expressions are in a form most suitable for the discussion of the tide observations and comparisons with theory, and for the study and investigation of the tidal theory, yet on account of the great number of arguments which would have to be used, and number of terms taken in, resulting from the developments, the whole result can be put into a more suitable form for prediction, containing but few arguments. For this purpose we shall determine from the constants belonging to the resultant tides of the moon and sun, which have been obtained directly from comparisons of observation with the tidal expressions, certain constants belonging to the lunar and solar tides considered separately, and then combine these separate tides into a form of expression similar to that of the resultant expression of the potentials of the disturbing forces of the moon and sun, (5) and (6).

By neglecting the effect of friction depending upon F, which decreases the larger tides a little more in proportion than the smaller ones, the first of (25) may be used, with the value of E which we have obtained, in determining the relative magnitudes of the lunar and solar tides. In this case the quantity corresponding to U_i is $D_t\,\eta_1 = .426$, twice the difference of the velocities of the moon and sun in right ascension. If we therefore put e' = the ratio between the solar and lunar tide, since e (8) is the ratio between the solar and lunar disturbing forces, we shall have

$$e' = e(1 - .426\,E) = 0.180 \tag{92}$$

using the values of e, $\delta\mu$, and E in (10) and (90). Hence the solar tide is decreased in the ratio of 1 to 1—.426 E, or of 1 to .401, in consequence of the sun's having a slower motion in right ascension than the moon.

44. If we now consider the lunar tide alone, omitting that part of the effect of friction which depends upon F, since in this case the inequalities in the disturbing force depend upon the moon's parallax and declination only, we shall have in (17)

$$\Sigma_i\,P_i \cos \eta_i = C\left(1 + \frac{\delta p}{p}\right)^3 (1 - \sin^2 v) \tag{93}$$

in which C is a constant and p is the mean parallax, δp is the excess of the parallax above the mean parallax, and v is the moon's declination, as heretofore defined. In this case the values of P_i in the first member, and the corresponding values of U_i, are different from those already given in the case of the resultants of the potentials, and are found to be

$$\left\{\begin{array}{ll} P_1 = .0250, & U_1 = -.0073 \\ P_2 = .1638, & U_2 = -.0500 \\ P_3 = .0860, & U_3 = .0397 \end{array}\right. \tag{94}$$

omitting the terms depending upon the moon's node and other small secondary terms. With these values the first of (25) gives, omitting F,

$$\left\{\begin{array}{l} R_2 = .1638 + .0503 \times 1.408 = .2342 \\ R_3 = .0860 - .0397 \times 1.408 = .0302 \end{array}\right. \tag{95}$$

Hence R_2, and consequently the coefficient of the corresponding tidal inequality, is increased in the ratio of .1638 to .2342, or of 1 to 1.429, in consequence of the moon's excess of motion at perigee over its mean motion. Also R_3, and the corresponding tidal coefficient, are decreased in the ratio of .0860 to .0302, or of 1 to .351, in consequence of the moon's decreased motion in right ascension when on the equator, on account of the obliquity of the ecliptic. It should be understood that the preceding ratios are not the ratios of increase or decrease compared with the whole tidal coefficient, but the ratio of increase or decrease of the whole tidal coefficient compared with the corresponding inequality having the same argument. The ratio of increase of the mean tidal coefficient at the maximum is, in the former case, $1+.0500\times1.408=1.070$, and of decrease in the latter, $1-.0397\times1.408=.944$.

Since the increase of the moon's angular motion is proportional to the increase of the parallax, for any increase of the parallax above the mean parallax the ratio of increase of the tidal coefficient of the inequality or variation is 1.429 times greater. If we also compare the moon's motion in right ascension with that motion when on the equator, the increase of motion may be assumed to be as $\sin^2 v$. This is not strictly true, especially with regard to the effects depending upon the moon's node, but the error, as applied to this small inequality, is insensible even in the Boston tides. For any decrease, therefore, of the coefficient of the disturbing force due to declination, the ratio of decrease of the corresponding tidal coefficient is .351.

What has been stated with regard to the lunar tide is also true of the solar tide, except that the relative ratios of increase or of decrease are different.

45. If we therefore put, supposing $F=0$,

$M=$the value of A_2, (33), in the case of the moon,
$S=$its value in the case of the sun,

we shall have, without sensible error,

$$(96)\qquad \begin{cases} M= K\left(1+m\,\dfrac{\delta p}{p}\right)^3(1-n\sin^2 v) \\ S=e'K\left(1+m'\dfrac{\delta p'}{p'}\right)^3(1-n'\sin^2 v') \end{cases}$$

in which the accented letters denote the same with regard to the sun which the same letters without an accent do with regard to the moon, and in which p and v must be taken at the times τ_2 and τ_3 earlier. In the case of the moon we have found $m=1.429$, $n=.351$. On account of the sun's slow motion in right ascension, m' and n' may be put equal to unity without sensible error.

If we combine the separate expressions of the lunar and solar tides, as given by (33), as in the case of the potentials of the disturbing forces (5), we get for the combined semi-diurnal tides,

$$(97)\qquad Y_2=\sqrt{M^2+S^2+2MS\cos(\eta_1-a_1)}\cos(2\rho-l-\beta')=Q\cos(2\rho-l-\beta')$$

in which

$$(98)\qquad \tan\beta'=-\frac{S\sin(\eta_1-a_1)}{M+S\cos(\eta_1-a_1)}$$

These expressions do not include certain small corrections belonging to inequalities of a second order due to changes of the moon's motion in right ascension, but these are very small and of no practical importance.

The lunitidal interval in the preceding expression, in solar time, is

$$(99)\qquad L=1.035\frac{\beta'+l}{D_t(2\rho-l-\beta')}$$

and is equivalent to (27), putting $\Sigma_i\, Q_i=1.035\,\beta'$, and using the preceding values of P_i, belonging to the moon only, in the rest of the expression.

The great advantage of the preceding forms of expressions is that we dispense with all development in the computation of the tidal coefficient, and the trouble of taking into account a very great number of small terms, in the development with arguments depending upon the sums and differences of the principal arguments, and have as arguments merely the time of the moon's transit and the parallaxes and declinations of the moon and sun. The same is true also of the part of the lunitidal

interval depending upon β', which is the principal part, the remainder being generally quite small, so that it need be applied as a correction only to a few of the principal terms.

46. If we develop the preceding expressions, as in the case of the potential of the disturbing force, substituting for p, v, p', and v', their expressions depending upon the angles η_i, we should obtain expressions of the resultant of the lunar and solar tides similar to those in the preceding pages, (25) and (27), obtained from the resultant of lunar and solar disturbing forces, of which the constants should be equivalent, and the coefficients of the inequalities of the latter, divided by $(1+F)$, should be equal to the corresponding ones in the former.

In the comparison of the constants we get

(100) $$\sqrt{1+e'^2}\,.956\,K=K_2$$

The preceding must be used as a condition to determine K in (96).

In the comparison of the coefficients of the semi-monthly inequalities in the development of (97) and (98), using the value of e' (92) with those given by observation and by (25) and (27), it is found that the coefficient of the inequality in heights is too great by $0^m.7$, and that of the lunitidal intervals is too small by $1^m.5$. The theory with regard to this last expression and development seems to be in error by these amounts. But as our object here is merely to get the most convenient expression which will represent the observations with sufficient accuracy for practical purposes, this can be obtained by changing a little the constants as obtained from the former conditions. If we take

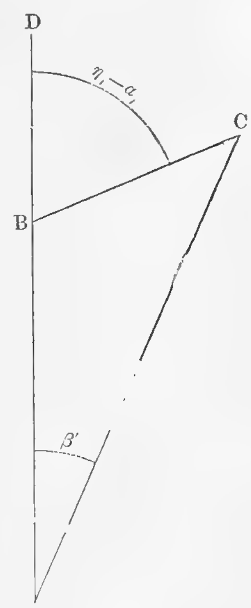
(101) $$e'=.205, \qquad m=1.530, \qquad n=.410, \qquad F=.500$$

with these values in (96), (97) will give the principal inequalities in the heights, except the effects of F, within $0^m.2$, and all the others with about the same accuracy as (25), and (97) will give such a value of β' as, substituted in (99), will give the coefficients of the intervals within $0^m.5$, except the few discrepancies already mentioned in the comparisons with the expression of (27). The preceding constants, however, in this case do not quite have the relations to one another required by theory in the other developments.

With the preceding value of e', and the value of K_2 (60), (100) gives for the constant in (96),

(102) $$K=5.004 \text{ feet.}$$

47. Having obtained M and S from (96), the coefficient of the tide Q and the value of β' in (97) and (98) are readily obtained by construction as follows:



Take A B equal to M, and B C equal to S, making an angle with A B, produced to D, equal to $\eta_1-\alpha_1$, which is twice $\psi-\psi'$ at a time τ_1, previous to the time of high or low water, and then join A C. The latter is Q (97), or the coefficient of the tide, neglecting the effect of F. The angle B A C also is the value of β'. Of course the same are readily obtained by a trigonometrical calculation. One-half β', reduced to time and increased by $\frac{1}{30}$th, is the part of the lunitidal interval in solar time depending upon β'.

The values of M and S (96) contain only the parallaxes and declinations of the moon and sun as arguments, and very simple and convenient tables may be constructed giving their values for any given arguments; and then by a very simple construction, or trigonometrical calculation, we get the coefficient of the tide and the principal part of the interval, and thus take in completely all the numerous terms arising from any developed form of expression. These terms do not consist only of the terms corresponding to the terms given in the development of the disturbing forces in (12) and (18), but a great many others of the same order as many given there. The coefficient of the tide thus obtained must be diminished by one-third of the inequality from the mean tide for the effect of friction depending upon F, which is the same as dividing the inequality by $(1+F)$.

It now remains to determine the part of L depending upon l in (99), which is the lunitidal interval of the lunar tide. The constant of this is B_0 determined by

observation (56), and the inequalities are determined by (27) omitting Q, and using the preceding values of P_i (94) belonging to the lunar tide only. This gives for the lunar part,

(103) $$M_1 = 1^m.7, \qquad M_2 = 5^m.8, \qquad M_3 = 6^m.1, \&c.$$

The first of these belongs to the small term in the moon's parallax depending upon variation, and is added to the value of the first inequality depending upon β', which is $-24^m.2$, to give the whole semi-monthly inequality $-22^m.5$. The second is the whole value of B_2, there being no part depending upon β'. The part of the third inequality depending upon β' is $-1^m.0$, and hence the whole value of B_3 is $5^m.1$.

48. For the sake of convenience in computation we can put in (27), in the case of the lunar tide,

(104) $$\Sigma_i M_i \sin \eta_i = C\, D_t\, p + C' \sin 2\, v\, D_t\, v$$

in which

$D_t\, p$ = the hourly change of parallax in seconds,
$D_t\, v$ = the change of declination in seconds for one minute of time.

For the principal term of parallax of which the coefficient is $186''.5$; the hourly change is $1''.79 \sin \eta_2$; also, the change of $\sin 2\, v\, D_t\, v$ in seconds for one minute of time is $5''.6 \sin \eta_3$. Hence, the constants in the preceding expression are determined by the following conditions, using $5^m.3$ given by observation for the value of M_i instead of $5^m.8$ given by theory, (103):

(105) $$\begin{cases} 1.79\, C = 5^m.3 \\ 5.6\ \ C' = 6^m.1 \end{cases}$$

These conditions give $C=3$ very nearly, and $C'=1.1$. With these constants, (104), using seconds of arc as minutes of time, gives the sum of all the terms depending upon $D_t\, \eta_i$, independently of any developed expression, directly from the hourly differences of parallax, and the differences of declination for one minute, taken from the Nautical Almanac.

In addition to the preceding we have the terms $\Sigma_i N_i \cos \eta_i$ (27) depending upon friction and other disturbing causes, of which it is only necessary to take account of the following, in which the coefficients given by observation are used, being reduced from transit D to the transit occurring at the time τ before the time of the tide, using the correction (34) for changing from one transit to another:

$$\begin{aligned} N_1 \cos \eta_1 &= 4^m.0 \cos \eta_1 \\ N_2 \cos \eta_2 &= -6^m.0 \cos \eta_2 \\ N_3 \cos \eta_3 &= 1^m.5 \cos \eta_3 \\ N_4 \cos \eta_4 &= 4^m.0 \cos \eta_4 \\ N_6 \cos \eta_6 &= 2^m.5 \cos \eta_6 \end{aligned}$$

All the other terms of this form are included in the terms depending upon β' in (99).

The first three and the last of these terms are embraced in Table III, the fourth one in Table II.

To both the times and heights must be added, also, the effect of the term depending upon the fourth power of the moon's distance, given in (69) and (70). These inequalities are given in the last two columns of Table IV.

The summation of the preceding effects gives the values of A_2, (24) or (33), and L_2, (26).

The value of H_0, the height of mean level, neglecting the very small inequalities given by observation as of no practical importance, is given in Table III.

The value of A_2 added to H_0, gives the height of high water, and, subtracted from H_0, gives that of low water.

To both the heights and times must then be added the effects of the lunar and solar diurnal tides to obtain the complete height and time of the tide. The effects of the lunar tide upon both the height and time of the tide are contained in Table IV, and those of the solar diurnal tide in Tables VIII, IX, X, and XI.

COMPUTATION OF A TIDAL EPHEMERIS.

49. The method of using the preceding formulæ and results in the computation of a tidal ephemeris is most conveniently explained by a reference to the example given at the end:

A is the mean time of the moon's upper transit over the meridian preceding the Washington meridian 2.4 hours, and is obtained from page 332 of the American Ephemeris by interpolation by means of the hourly differences.

a is the equation of time to be subtracted from mean time.

B contains the hours and minutes of the apparent time of the moon's transit over the meridian above stated.

C is the moon's horizontal parallax for a time 18 hours before the time of the Washington transit, or about 6 hours after the preceding upper transit, taken from page 339 of the American Ephemeris, by interpolation by means of the hourly differences.

c is the corresponding hourly difference.

D is the moon's declination 7 hours preceding the time of Washington transit, or 2 hours preceding the time of the Greenwich transit, taken from page 6 of the American Ephemeris.

d is the corresponding hourly difference of declination.

L′ is the part of L (99) depending upon β', or upon the solar tide, plus a constant of 30 minutes.

The part independent of the constant, and also Q, can be readily obtained, with sufficient accuracy for practical purposes, by construction or by trigonometry, as has been explained in (§47). But the method by trigonometry does not answer well near the times of the conjunctions or quadratures, where the angles are very small, unless these angles are determined with great accuracy. If L′ and Q are determined by computation it is best to compute Q and β', (97) and (98), directly from the expressions, as in the last part of the example at the end, in which

M is taken from Table I, with the arguments C and D, and

S from Table II, with the date as the argument.

Log sin 2 B and log cos 2 B are taken from Table VII, which is so arranged, with the sine and cosine adjacent to each other, that they can be taken out at the same time, using B instead of 2 B as an argument. The remaining steps in the example, to obtain tan β', need no explanation. With tan β' as an argument, the variable part of L′ is taken from Table VI, to which the constant, 30 minutes, is added to make all the values positive.

e is equal to three times c with the sign changed, calling seconds of arc minutes of time, plus a constant of 10 minutes.

f is taken from Table V, with D and d as arguments, a constant of 10 minutes being also added.

g is taken from Table III, with the time of transit, B, as an argument.

h is taken from the same, with the parallax, C, as an argument.

i is also taken from the same, with the declination, D, as an argument.

j is taken from the last part of Table IV, using the declination, D, one day in advance, as taken out above.

B_0 is taken from Table II, with the date as the argument, the day and hours not being written in the example, but borne in mind.

$E = A + L' + e + f + g + h + i + j + B_0$

The value of A being taken from the ephemeris in astronomical time, the constant, B_0, in the table has been increased 12 hours in order to give E in civil time. When the *apparent* time of high water is required, B should be used instead of A in the preceding expression of E.

Δ^1 and Δ^2 are the first and second differences of E belonging to the upper transits, used as a check, and also for interpolating the intermediate numbers belonging to the lower transits. The minutes only of Δ^1 are written in the example, the 24 hours being understood. The intermediate numbers, in smaller type, are $\frac{1}{2}\Delta^1$ and $\frac{1}{8}\Delta^2$, used in the interpolation.

δ^1 are the differences after interpolation, used as a check, and also in interpolating to low water, the 12 hours understood not being written.

k is taken from the last part of Table IV, with D as an argument taken one day in advance.

l is the effect of the solar diurnal tide upon the time of high water, taken from Table VIII, with B and the day of the year as arguments.

$m = k + l$ is the effect of the whole diurnal tide upon the time of high water. The values belonging to the lower transit are readily obtained by interpolation, and must be used with a contrary sign.

$t.h.w=\mathrm{E}+m$ are the times of high water; the days and hours, being the same as in E, are not written.

Q, the coefficient of the semi-diurnal tide, independent of the effect of F, is obtained either by construction, as has been explained, or from log Q in the latter part of the example, when obtained by computation. The different steps in the example, by which log Q is obtained, need no explanation.

$n=\mathrm{Q}-4.90$ ft.; that is, it is the excess of Q above its mean value. One-third of this subtracted from Q gives $\mathrm{A}_2=4.90+\frac{\mathrm{Q}-4.90}{1+\mathrm{F}}$, or the coefficient of the semi-diurnal tide as affected by F.

Δ^1, Δ^2 are the first and second differences of A_2, used as a check, and also in interpolating for the lower transits.

δ^1 are the first differences of A_2 after interpolation, used as a check, and for interpolating to low water.

$\mathrm{H}_0+\mathrm{A}_2$, in which H_0 is taken from Table II, with the day of the year as an argument, is the height of high water of the semi-diurnal tide.

p is taken from the first part of Table IV, with the declination, D, one day in advance, and is the effect of the lunar diurnal tide upon the height of high water.

q is taken from Table X, with B and the day of the year as arguments, and is the effect of the solar diurnal tide upon the height of high water above the zero of the tide-gauge.

$r=p+q$ is the effect of the whole diurnal tide upon the height of high water. The values of r for the lower transits are readily obtained by interpolation, and must be used with the contrary sign.

$h.h.w=\mathrm{H}_0+\mathrm{A}_2+r$ is the height of high water.

E′ is obtained from E by interpolation to the time of low water by means of δ^1, using only the first differences.

k' is taken from the first part of Table IV for low water, using the argument D one day in advance.

l' is taken from Table IX, with the arguments B and the day of the year.

$m'=k'+l'$ is the whole effect of the diurnal tide upon the times of low water. The values of m for lower transits are obtained by interpolation and must be used with the contrary sign.

$t.l.w=\mathrm{E}+m'$ are the times of low water, the days and hours being the same as in E′.

A'_2 is obtained from A_2 by interpolation to the time of low water.

$\mathrm{H}_0-\mathrm{A}'_2$ is the height of the low water of the semi-diurnal tide.

p' is taken from the first part of Table IV for low water, using the argument D one day in advance, and is the effect of the lunar diurnal tide upon the height of low water.

q' is taken from Table XII, with B and the day of the year as arguments, and is the effect of the solar diurnal tide upon the height of low water.

$r'=p'+q'$ is the effect of the whole diurnal tide upon the height of low water. The values of r for lower transits are obtained by interpolation, and must have the contrary sign.

$h.l.w=\mathrm{H}_0-\mathrm{A}'_2+r$ is the height of low water.

CONCLUSION.

50. In the preceding discussion all the results have been brought out which can be of much interest to any one in any theoretical tidal investigations; and much attention has been given to the arranging and presenting of the whole matter in as systematic and concise a manner as possible, and with a convenient and appropriate notation. These results must be found to be the more interesting to investigators on account of the great peculiarities due to local circumstances which make them differ so much in many respects from the results obtained in most of European ports. A comparison of these results has also been made with both the equilibrium and the dynamical theories of the tides, so far as it could be done, where it is neither convenient nor proper to enter very thoroughly into the discussion of tidal theories; and the great defects of the equilibrium theory have been shown, and also various discrepancies of a small order between the results and

our tidal expressions based upon the dynamic theory have been pointed out. These are due, no doubt, mostly to friction in connection with ocean currents, and to the peculiarities arising from local circumstances, which cause many small terms, which are necessarily neglected in the tidal expressions, and which in most ports are insensible, to be quite large in the Boston tides.

Much study and care has also been given to the formation, from these results, of the most convenient formulæ possible for the prediction of the times and heights of the tides, and by means of various auxiliary tables, to render the labor of their computation as small as possible. An example has also been given of the most convenient method of carrying out the computations, from which it may be seen that they can be made with great facility and also with great accuracy.

In the comparison of individual tides as computed with the formulæ and tables, with observation, considerable discrepancies are often found in both the times and the heights, as is to be expected, on account of the many abnormal disturbances arising from the changes in the forces and directions of the wind and in the barometric pressure, and these discrepancies are especially found during the winter and spring, when these changes are the greatest; but still it is thought that the computation gives very accurately the true normal tide. From the discussion of these residuals with reference to the winds and the barometric pressure, some interesting results may yet be obtained with regard to their effects upon the tides. The computation of the tides for a portion of the series is now being made for this purpose, the results of which must be the subject of a future report.

In the prosecution of the preceding discussions I have to acknowledge the receipt of much valuable and very satisfactory aid from the Misses Lane, in the Coast Survey service.

Very respectfully, yours,

WM. FERREL.

Professor BENJAMIN PEIRCE,

Superintendent United States Coast Survey.

NOTE.—The following tables are added by way of appendix to the preceding discussion of the Boston tides. They were prepared by Mr. Ferrel, and show the application of the theory.

APPENDIX TO THE DISCUSSION OF THE BOSTON TIDES, BY MR. W. FERREL.

TABLE I—*Showing the value of M for every 10″ of the moon's parallax, and for every 2° of its declination.*

Moon's parallax.		Moon's declination.														
		0 . . . 2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°
′	″	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*
54	0	3.88	3.87	3.86	3.85	3.83	3.81	3.79	3.76	3.73	3.70	3.66	3.62	3.58	3.53	3.48
	10	3.94	3.93	3.92	3.90	3.88	3.86	3.84	3.81	3.78	3.75	3.71	3.67	3.63	3.58	3.53
	20	4.00	3.99	3.98	3.96	3.94	3.92	3.90	3.87	3.84	3.81	3.77	3.73	3.69	3.64	3.58
	30	4.06	4.05	4.04	4.02	4.00	3.98	3.96	3.93	3.90	3.86	3.82	3.78	3.74	3.69	3.64
	40	4.12	4.11	4.10	4.08	4.06	4.04	4.02	3.99	3.96	3.91	3.87	3.83	3.79	3.74	3.69
	50	4.18	4.17	4.16	4.14	4.12	4.10	4.08	4.05	4.01	3.97	3.93	3.89	3.85	3.80	3.74
55	0	4.24	4.23	4.22	4.20	4.17	4.15	4.13	4.10	4.07	4.03	3.99	3.95	3.90	3.85	3.79
	10	4.30	4.29	4.28	4.26	4.23	4.21	4.19	4.16	4.13	4.09	4.05	4.00	3.95	3.90	3.85
	20	4.36	4.35	4.34	4.32	4.29	4.27	4.25	4.22	4.18	4.14	4.10	4.05	4.00	3.95	3.90
	30	4.42	4.41	4.40	4.38	4.36	4.34	4.31	4.28	4.24	4.20	4.16	4.11	4.06	4.01	3.96
	40	4.48	4.47	4.46	4.44	4.42	4.40	4.37	4.34	4.30	4.26	4.22	4.17	4.12	4.07	4.01
	50	4.54	4.53	4.52	4.50	4.48	4.46	4.43	4.40	4.36	4.32	4.28	4.23	4.17	4.12	4.06
56	0	4.60	4.59	4.58	4.56	4.54	4.52	4.49	4.46	4.42	4.37	4.32	4.27	4.22	4.17	4.12
	10	4.67	4.66	4.65	4.63	4.60	4.58	4.55	4.51	4.47	4.43	4.38	4.33	4.28	4.23	4.17
	20	4.74	4.73	4.72	4.70	4.67	4.65	4.62	4.58	4.54	4.50	4.45	4.40	4.35	4.29	4.23
	30	4.81	4.80	4.79	4.77	4.74	4.72	4.69	4.65	4.61	4.56	4.51	4.46	4.41	4.35	4.29
	40	4.87	4.86	4.85	4.83	4.81	4.79	4.76	4.72	4.67	4.62	4.57	4.52	4.47	4.41	4.35
	50	4.93	4.92	4.91	4.89	4.87	4.85	4.82	4.78	4.73	4.68	4.63	4.58	4.53	4.47	4.41
57	0	5.00	4.99	4.98	4.96	4.94	4.92	4.89	4.85	4.80	4.75	4.70	4.65	4.59	4.53	4.47
	10	5.07	5.06	5.05	5.03	5.00	4.98	4.95	4.91	4.86	4.81	4.76	4.71	4.66	4.60	4.54
	20	5.14	5.13	5.12	5.10	5.07	5.05	5.02	4.98	4.93	4.87	4.82	4.77	4.72	4.66	4.60
	30	5.21	5.20	5.19	5.17	5.14	5.12	5.09	5.05	5.00	4.94	4.89	4.84	4.79	4.73	4.67
	40	5.28	5.27	5.26	5.24	5.21	5.19	5.16	5.11	5.06	5.01	4.96	4.91	4.85	4.79	4.73
	50	5.35	5.34	5.33	5.30	5.27	5.25	5.22	5.18	5.13	5.08	5.03	4.97	4.91	4.85	4.79
58	0	5.41	5.40	5.39	5.37	5.34	5.32	5.29	5.25	5.20	5.14	5.09	5.03	4.97	4.91	4.85
	10	5.48	5.47	5.46	5.44	5.41	5.39	5.36	5.32	5.27	5.21	5.16	5.11	5.05	4.98	4.91
	20	5.55	5.54	5.53	5.51	5.49	5.47	5.44	5.40	5.34	5.28	5.23	5.17	5.11	5.05	4.98
	30	5.63	5.62	5.61	5.59	5.56	5.54	5.51	5.46	5.41	5.35	5.29	5.23	5.17	5.11	5.04
	40	5.70	5.69	5.68	5.67	5.64	5.61	5.57	5.52	5.47	5.42	5.37	5.31	5.24	5.17	5.10
	50	5.77	5.76	5.75	5.73	5.71	5.68	5.65	5.61	5.55	5.49	5.43	5.37	5.31	5.24	5.17
59	0	5.85	5.84	5.83	5.81	5.78	5.75	5.72	5.68	5.62	5.56	5.50	5.44	5.37	5.30	5.23
	10	5.93	5.92	5.90	5.88	5.85	5.82	5.78	5.73	5.68	5.63	5.57	5.51	5.44	5.37	5.30
	20	6.00	5.99	5.97	5.95	5.92	5.89	5.85	5.81	5.76	5.71	5.65	5.58	5.51	5.44	5.37
	30	6.08	6.07	6.06	6.03	6.00	5.97	5.93	5.89	5.84	5.78	5.72	5.65	5.58	5.51	5.43
	40	6.15	6.14	6.13	6.10	6.07	6.04	6.00	5.95	5.90	5.85	5.79	5.72	5.65	5.58	5.50
	50	6.23	6.22	6.20	6.17	6.14	6.11	6.07	6.02	5.97	5.92	5.86	5.79	5.72	5.64	5.56
60	0	6.30	6.29	6.28	6.25	6.22	6.19	6.15	6.10	6.05	5.99	5.92	5.85	5.78	5.71	5.63
	10	6.38	6.37	6.36	6.33	6.30	6.27	6.23	6.18	6.12	6.06	5.99	5.92	5.85	5.78	5.70
	20	6.46	6.45	6.43	6.41	6.38	6.35	6.31	6.26	6.20	6.14	6.07	6.00	5.93	5.85	5.77
	30	6.54	6.53	6.51	6.49	6.46	6.43	6.39	6.34	6.28	6.22	6.15	6.08	6.00	5.92	5.84
	40	6.62	6.61	6.60	6.57	6.54	6.51	6.47	6.42	6.36	6.30	6.23	6.16	6.08	6.00	5.92
	50	6.70	6.69	6.67	6.65	6.62	6.59	6.55	6.50	6.44	6.37	6.30	6.23	6.15	6.07	5.99
61	0	6.79	6.78	6.76	6.73	6.70	6.67	6.63	6.58	6.52	6.45	6.38	6.31	6.23	6.15	6.07
	10	6.88	6.87	6.85	6.82	6.78	6.75	6.71	6.66	6.60	6.53	6.46	6.39	6.31	6.23	6.15
	20	6.97	6.96	6.94	6.91	6.87	6.84	6.80	6.75	6.68	6.61	6.54	6.47	6.39	6.31	6.23
	30	7.05	7.04	7.02	6.99	6.95	6.91	6.87	6.82	6.76	6.70	6.63	6.55	6.47	6.39	6.37

TABLE II—*Showing the values of* S *and* H_0 *for each third of a month, and also of* B_0, *as affected by the annual inequality, and the constants of the equations.*

Date.	S	H_0	B_0
	Ft.	*Ft.*	*d. h. m.*
Jan. 1	0.92	20.09	1 23 9.0
11	0.93	20.07	9.1
21	0.95	20.04	9.3
Feb. 1	0.97	20.00	9.6
11	1.00	19.96	10.0
21	1.02	19.96	10.5
Mar. 1	1.03	19.98	11.0
11	1.03	20.01	11.6
21	1.02	20.03	12.3
April 1	1.01	20.06	13.0
11	1.00	20.08	13.7
21	0.97	20.11	14.4
May 1	0.93	20.14	15.0
11	0.90	20.15	15.5
21	0.87	20.16	15.9
June 1	0.84	20.16	16.3
11	0.83	20.16	16.6
21	0.81	20.16	16.9
July 1	0.82	20.17	17.0
11	0.83	20.17	16.9
21	0.86	20.18	16.6
Aug. 1	0.89	20.20	16.3
11	0.92	20.21	15.9
21	0.95	20.21	15.5
Sept. 1	0.98	20.22	15.0
11	1.00	20.23	14.4
21	1.01	20.25	13.7
Oct. 1	1.01	20.27	13.0
11	1.01	20.29	12.3
21	1.00	20.30	11.6
Nov. 1	0.98	20.31	11.0
11	0.96	20.31	10.5
21	0.94	20.30	10.0
Dec. 1	0.92	20.27	9.6
11	0.91	20.22	9.3
21	0.92	20.16	1 23 9.1

NOTE.—For the value of H_0 above mean low water, subtract 15.26 feet.

TABLE III—*Showing the inequalities resulting from friction and other causes, and depending upon the moon's transit, parallax, and declination.*

☽'s transit.	Equation.	☽'s parallax.	Equation.	☽'s dec.	Equation.
h. m.	*m.*	′ ″	*m.*	°	*m.*
0 00	8.0	54 00	15.0	0	0.0
30	7.8	30	14.0	2	0.1
1 00	7.5	55 00	13.0	4	0.2
30	6.9	30	12.0	6	0.4
2 00	6.0	56 00	11.0	8	0.6
30	5.0	30	10.0	10	0.9
3 00	4.0	57 00	9.0	12	1.2
30	3.0	30	8.0	14	1.6
4 00	2.0	58 00	7.0	16	2.0
30	1.1	30	6.0	18	2.5
5 00	0.5	59 00	5.0	20	3.0
30	0.2	30	4.0	22	3.6
6 00	0.0	60 00	3.0	24	4.3
30	0.2	30	2.0	26	5.1
7 00	0.5	61 00	1.0	28	6.0
30	1.1	30	0.0	30	7.0
8 00	2.0				
30	3.0				
9 00	4.0				
30	5.0				
10 00	6.0				
30	6.9	Constant 9*m*.0		Constant.... 2*m*.0	
11 00	7.5				
30	7.8				
Constant 4*m*.0					

TABLE IV—*Showing the effect of the moon's diurnal tide upon the times and heights, and also of the term depending upon the fourth power of the moon's distance, contained in the last columns. The argument is* D, *taken one day in advance.*

☽'s dec.	Equations of high water.		Equations of low water.		Equations of semi-diurnal tide.	
°	*m.*	*Ft.*	*m.*	*Ft.*	*m.*	*Ft.*
+30	−3.0	+0.63	+3.0	−0.37	0.0	0.06
25	2.5	0.53	2.5	0.32	0.3	0.06
20	2.0	0.43	2.0	0.26	0.8	0.05
15	1.5	0.32	1.5	0.20	1.4	0.05
10	1.0	0.22	1.0	0.13	1.9	0.04
+ 5	−0.5	+0.11	+0.5	−0.07	2.5	0.04
0	0.0	0.00	0.0	0.00	3.0	0.03
− 5	+0.5	−0.11	−0.5	+0.07	3.5	0.02
10	1.0	0.22	1.0	0.13	4.1	0.02
15	1.5	0.32	1.5	0.20	4.6	0.01
20	2.0	0.43	2.0	0.26	5.2	0.01
25	2.5	0.53	2.5	0.32	5.7	0.00
−30	+3.0	−0.63	−3.0	+0.37	6.0	0.00

NOTE.—For lower transits the signs of the diurnal tide must be reversed.

TABLE V—*Showing the value of* C′ sin 2 v D_t v (104) *in tenths of minutes for each degree of declination and for each second of the value of* D_t v.

Dec.	Values of D_t v.																
	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″	12″	13″	14″	15″	16″	17″
°																	
1	.	.	.	.	.	.	.	3	3	4	4	5	5	5	6	6	7
2	.	.	.	.	.	.	.	6	7	8	8	9	10	11	12	12	13
3	.	.	.	.	.	.	.	9	10	11	12	13	15	16	17	18	19
4	.	.	.	.	.	..	..	12	13	15	16	18	20	21	23	25	26
5	.	.	.	.	..	..	..	15	16	19	20	22	25	26	29	31	32
6	.	.	.	.	..	..	16	18	20	23	25	27	30	32	34	37	40
7	.	.	.	..	..	..	19	21	23	27	29	31	35	37	40	43	46
8	.	.	..	..	..	..	22	24	27	30	33	36	40	42	46	49	52
9	.	.	..	..	..	20	24	27	30	34	37	41	44	47	51	54	..
10	.	.	..	..	..	22	26	30	33	37	41	46	49	53	57	60	..
11	.	.	..	..	..	25	29	33	37	40	45	50	54	58	62	66	..
12	.	..	..	..	22	27	31	36	40	44	49	54	58	63	67	72	..
13	.	..	..	..	24	29	34	39	43	47	53	58	63	68	72	78	..
14	.	..	..	20	25	31	36	42	46	51	57	62	67	72	77	..	..
15	.	..	16	22	27	33	38	44	49	55	60	66	71	76	82	..	..
16	..	12	17	24	28	35	41	47	52	58	64	70	76	81	86	..	..
17	6	12	18	25	30	37	43	50	55	61	68	74	80	86	..	..	..
18	6	13	19	26	32	39	45	52	58	64	71	78	84	90	..	..	..
19	6	13	20	27	33	40	47	55	60	67	75	82	86	..	..	..	..
20	7	14	21	28	35	42	49	57	63	70	78	86	90	..	..	..	..
21	7	15	22	29	36	44	51	59	66	73	81	90	..	..	..	..	..
22	7	15	23	30	37	46	53	61	69	76	84	..	..	..	..	..	..
23	8	16	23	31	39	47	55	63	71	79	..	..	..	..	..	..	..
24	8	16	24	32	40	49	57	65	73	82	..	..	..	..	..	..	..
25	8	17	25	33	41	50	59	67	75	..	..	..	..	..	..	..	..
26	8	17	26	34	43	52	61	69	..	..	..	..	..	..	..	..	..
27	9	18	27	35	44	53	63	..	..	..	..	..	..	..	..	..	..
28	9	18	27	36	45	54	..	..	..	..	..	..	..	..	..	..	..
29	9	19	28	37	46	..	..	..	..	..	..	..	..	..	..	..	..

NOTE.—When the arguments have the same signs, the quantities are positive; when different signs, negative.

TABLE VI—*Showing the value of* L′, *and the part of* L (99) *depending upon* β′, *corresponding to any given value of the logarithmic tangent of* β′ ( )

L′	0*m.*	1*m.*	2*m.*	3*m.*	4*m.*	5*m.*	6*m.*	7*m.*	8*m.*	9*m.*
m.										
0	. . . .	6. 926	7. 227	7. 403	7. 528	7. 615	7. 704	7. 771	7. 829	7. 880
1	7. 926	7. 967	8. 005	8. 040	8. 072	8. 101	8. 130	8. 156	8. 181	8. 204
2	8. 227	8. 248	8. 268	8. 288	8. 306	8. 324	8. 341	8. 357	8. 373	8. 388
0	. . . .	7. 926	8. 227	8. 403	8. 523	8. 625	8. 704	8. 771	8. 830	8. 881
10	8. 927	8. 969	9. 006	9. 042	9. 074	9. 104	9. 133	9. 159	9. 185	9. 209
20	9. 231	9. 253	9. 274	9. 293	9. 312	9. 333	9. 348	9. 365	9. 382	9. 398
30	9. 413	9. 428	9. 442	9. 457	9. 470	9. 484	9. 497	9. 510	9. 522	9. 534

NOTE.—In the first division of the table 0*m.*, 1*m.*, 2*m.*, &c., must be taken as tenths of a minute.

TABLE VII—*Showing the logarithmic sine and cosine of* $2(\psi-\psi')$ *to three places for each minute of* $(\psi-\psi')$.

$(\psi-\psi')$	Sine.	Cosine.	$(\psi-\psi')$	$(\psi-\psi')$	Sine.	Cosine.	$(\psi-\psi')$	$(\psi-\psi')$	Sine.	Cosine.	$(\psi-\psi')$
h. m.			*h. m.*	*h. m.*			*h. m.*	*h. m.*			*h. m.*
0 0	....	10.000	5 60	1 0	9.699	9.938	4 60	2 0	9.938	9.699	3 60
1	7.941	000	59	1	705	935	59	1	940	692	59
2	8.242	000	58	2	712	933	58	2	942	686	58
3	418	000	57	3	718	931	57	3	944	679	57
4	543	000	56	4	724	928	56	4	946	672	56
5	640	000	55	5	730	926	55	5	948	664	55
6	719	9.999	54	6	736	924	54	6	950	657	54
7	786	999	53	7	742	921	53	7	952	650	53
8	844	999	52	8	748	919	52	8	954	642	52
9	895	999	5 51	9	753	916	4 51	9	955	634	3 51
0 10	8.940	9.998	50	1 10	9.759	9.913	50	2 10	9.957	9.626	50
11	982	998	49	11	764	911	49	11	959	618	49
12	9.019	998	48	12	769	908	48	12	961	609	48
13	054	997	47	13	774	905	47	13	962	601	47
14	086	997	46	14	779	902	46	14	964	592	46
15	116	996	45	15	784	899	45	15	966	583	45
16	144	996	44	16	789	897	44	16	967	574	44
17	170	995	43	17	794	894	43	17	969	564	43
18	194	995	42	18	799	890	42	18	970	554	42
19	218	994	5 41	19	804	887	4 41	19	972	544	3 41
0 20	9.240	9.993	40	1 20	9.808	9.884	40	2 20	9.973	9.534	40
21	260	993	39	21	813	881	39	21	974	523	39
22	281	992	38	22	817	878	38	22	976	513	38
23	300	991	37	23	821	874	37	23	977	501	37
24	318	990	36	24	826	871	36	24	978	490	36
25	335	990	35	25	830	868	35	25	979	478	35
26	352	989	34	26	834	864	34	26	981	466	34
27	368	988	33	27	838	861	33	27	982	453	33
28	384	987	32	28	842	857	32	28	983	440	32
29	399	986	5 31	29	846	853	4 31	29	984	427	3 31
0 30	9.413	9.985	30	1 30	9.849	9.849	30	2 30	9.985	9.413	30
31	427	984	29	31	853	846	29	31	986	399	29
32	440	983	28	32	857	842	28	32	987	384	28
33	453	982	27	33	861	838	27	33	988	368	27
34	466	981	26	34	864	834	26	34	989	352	26
35	478	979	25	35	868	830	25	35	990	335	25
36	490	978	24	36	871	826	24	36	990	318	24
37	501	977	23	37	874	821	23	37	991	300	23
38	513	976	22	38	878	817	22	38	992	281	22
39	523	974	5 21	39	881	813	4 21	39	993	260	3 21
0 40	9.534	9.973	20	1 40	9.884	9.808	20	2 40	9.993	9.240	20
41	544	972	19	41	887	804	19	41	994	218	19
42	554	970	18	42	890	799	18	42	995	194	18
43	564	969	17	43	894	794	17	43	995	170	17
44	574	967	16	44	897	789	16	44	996	144	16
45	583	966	15	45	899	784	15	45	996	116	15
46	592	964	14	46	902	779	14	46	997	086	14
47	601	962	13	47	905	774	13	47	997	054	13
48	609	961	12	48	908	769	12	48	998	019	12
49	618	959	5 11	49	911	764	4 11	49	998	8.982	3 11
0 50	9.626	9.957	10	1 50	9.913	9.759	10	2 50	9.998	8.940	10
51	634	955	9	51	916	753	9	51	999	895	9
52	642	954	8	52	919	748	8	52	999	844	8
53	650	952	7	53	921	742	7	53	999	786	7
54	657	950	6	54	924	736	6	54	999	719	6
55	664	948	5	55	926	730	5	55	10.000	640	5
56	672	946	4	56	928	724	4	56	000	543	4
57	679	944	3	57	931	718	3	57	000	418	3
58	686	942	2	58	933	712	2	58	000	242	2
59	692	940	1	59	935	705	1	59	000	7.941	1
60	9.699	9.938	5 0	1 60	9.938	9.699	4 0	2 60	10.000	....	3 0

TABLE VIII—*Showing the effect of the solar diurnal tide upon the time of high water for every hour of the moon's transit, and for the first of each month, expressed in tenths of a minute.*

Month.	HOURS OF MOON'S TRANSIT IN ASTRONOMICAL TIME.																						
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Jan.....	+13	+14	+15	+15	+15	+14	+13	+11	+ 6	0	- 6	-11	-13	-14	-15	-15	-15	-14	-13	-11	- 6	0	+ 6	+11
Feb.....	10	12	12	12	12	12	10	8	4	0	4	8	10	12	12	12	12	12	10	8	5	0	5	8
Mar....	+ 4	+ 5	+ 5	+ 5	+ 5	+ 5	+ 4	+ 3	+ 2	0	- 2	- 3	- 4	- 5	- 5	- 5	- 5	- 5	- 4	- 3	- 2	0	+ 2	+ 3
Apr.....	- 2	- 3	- 3	- 3	- 3	- 3	- 2	- 2	- 1	0	+ 1	+ 2	+ 2	+ 3	+ 3	+ 3	+ 3	+ 3	+ 2	+ 2	+ 1	0	- 1	- 2
May....	9	10	10	10	10	9	7	6	3	0	3	6	7	9	10	10	10	10	9	7	4	0	4	7
June....	12	14	14	14	14	13	12	10	5	0	5	10	12	13	14	14	14	14	12	10	6	0	6	10
July....	13	15	15	15	15	15	14	11	6	0	6	11	14	15	15	15	15	15	13	11	6	0	6	11
Aug....	9	12	12	12	12	12	11	8	4	0	4	8	11	12	12	12	12	12	10	8	5	0	4	8
Sept....	- 3	- 5	- 5	- 5	- 5	- 5	- 4	- 3	- 2	0	+ 2	+ 3	+ 4	+ 5	+ 5	+ 5	+ 5	+ 5	+ 4	+ 3	+ 2	0	- 2	- 3
Oct.....	+ 2	+ 3	+ 3	+ 3	+ 3	+ 3	+ 2	+ 2	+ 1	0	- 1	- 2	- 2	- 3	- 3	- 3	- 3	- 3	- 2	- 2	- 1	0	+ 1	+ 2
Nov....	9	10	10	10	10	10	8	6	3	0	3	6	8	10	10	10	10	10	9	7	4	0	4	7
Dec.....	+12	+14	+14	+14	+14	+13	+12	+10	+ 5	0	- 5	-10	-12	-13	-14	-14	-14	-13	-12	-10	- 6	0	+ 6	+10

NOTE.—For lower transits the signs must be reversed.

TABLE IX—*Showing the effect of the solar diurnal tide upon the time of low water for every hour of the moon's transit, and for the first of each month, expressed in tenths of a minute.*

Month.	HOURS OF MOON'S TRANSIT IN ASTRONOMICAL TIME.																						
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Jan.....	-13	-11	- 6	0	+ 6	+11	+13	+14	+15	+15	+15	+14	+13	+11	+ 6	0	- 6	-11	-13	-14	-15	-15	-15	-14
Feb.....	9	8	4	0	4	8	9	10	12	12	12	10	9	7	4	0	4	7	9	10	12	12	12	10
Mar....	- 3	- 3	- 2	0	+ 2	+ 3	+ 3	+ 4	+ 5	+ 5	+ 5	+ 5	+ 4	+ 3	+ 2	0	- 2	- 3	- 4	- 5	- 5	- 5	- 5	- 4
April...	+ 2	+ 2	+ 1	0	- 1	- 2	- 2	- 3	- 3	- 3	- 3	- 3	- 2	- 2	- 1	0	+ 1	+ 2	+ 2	+ 3	+ 3	+ 3	+ 3	+ 3
May....	9	7	3	0	3	7	9	9	10	10	10	9	8	7	3	0	3	7	8	9	10	10	10	9
June....	12	10	6	0	6	10	12	12	14	14	14	13	12	11	5	0	5	11	12	13	14	14	14	12
July....	13	11	6	0	6	11	13	14	15	15	15	14	13	11	6	0	6	11	13	14	15	15	15	14
Aug....	9	8	4	0	4	8	9	10	12	12	12	10	9	8	4	0	4	8	9	10	12	12	12	10
Sept....	+ 3	+ 3	+ 2	0	- 2	- 3	- 3	- 4	- 5	- 5	- 5	- 5	- 4	- 3	- 2	0	+ 2	+ 3	+ 4	+ 5	+ 5	+ 5	+ 5	+ 4
Oct.....	- 2	- 2	- 1	0	+ 1	+ 2	+ 2	+ 3	+ 3	+ 3	+ 3	+ 3	+ 2	+ 2	+ 1	0	- 1	- 2	- 2	- 3	- 3	- 3	- 3	- 3
Nov....	9	7	3	0	3	7	9	9	10	10	10	9	9	7	3	0	3	7	9	9	10	10	10	9
Dec.....	-12	-11	- 6	0	+ 6	+11	+12	+12	+14	+14	+14	+13	+12	+10	+ 5	0	- 5	-10	-12	-13	-14	-14	-14	-12

NOTE.—For lower transits the signs must be reversed.

TABLE X—*Showing the effect of the solar diurnal tide upon the height of high water for every hour of the moon's transit, and for the first of each month, expressed in hundredths of a foot.*

Month.	HOURS OF MOON'S TRANSIT IN ASTRONOMICAL TIME.																						
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Jan.....	-18	-13	- 6	0	+ 7	+13	+18	+22	+24	+24	+24	+22	+18	+13	+ 6	0	- 7	-13	-18	-22	-24	-24	-24	-22
Feb.....	13	10	4	0	5	10	13	16	18	19	18	16	13	10	4	0	5	10	13	16	18	19	18	16
Mar....	- 6	- 4	- 2	0	+ 2	+ 4	+ 6	+ 7	+ 8	+ 8	+ 7	+ 7	+ 6	+ 4	+ 2	0	- 2	- 4	- 6	- 7	- 8	- 8	- 7	- 7
April...	+ 3	+ 2	+ 1	0	- 1	- 2	- 4	- 4	- 5	- 5	- 4	- 4	- 3	- 2	- 1	0	+ 1	+ 2	+ 4	+ 4	+ 5	+ 5	+ 4	+ 4
May....	11	8	4	0	4	9	11	14	15	15	14	13	11	8	4	0	4	9	11	14	15	15	14	13
June....	16	11	5	0	6	12	16	20	22	23	22	21	16	11	5	0	6	12	16	20	22	23	22	21
July....	18	13	6	0	7	13	18	22	24	24	24	22	18	13	6	0	7	13	18	22	24	24	23	22
Aug....	13	10	4	0	5	10	13	16	18	19	18	16	13	10	4	0	5	10	13	16	18	19	18	16
Sept....	+ 6	+ 4	+ 2	0	- 2	- 4	- 6	- 7	- 8	- 8	- 7	- 7	- 6	- 4	- 2	0	+ 2	+ 4	+ 6	+ 7	+ 8	+ 8	+ 7	+ 7
Oct.....	- 3	- 2	- 1	0	+ 1	+ 2	+ 4	+ 4	+ 5	+ 5	+ 4	+ 4	+ 3	+ 2	+ 1	0	- 1	- 2	- 4	- 4	- 5	- 5	- 4	- 4
Nov....	11	8	4	0	4	9	11	14	15	15	14	13	11	8	4	0	4	9	11	14	15	15	14	13
Dec.....	-16	-11	- 5	0	+ 6	+12	+16	+20	+22	+23	+22	+21	+16	+11	+ 5	0	- 6	-12	-16	-20	-22	-23	-22	-21

NOTE.—For lower transits the signs must be reversed.

TABLE XI—*Showing the effect of the solar diurnal tide upon the height of low water for every hour of the moon's transit, and for the first of each month, expressed in hundredths of a foot.*

Month.	Hours of moon's transit in astronomical time.																							
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Jan	+11	+13	+14	+14	+13	+11	+ 9	+ 6	+ 3	0	— 4	— 8	—11	—13	—14	—14	—13	—11	— 9	— 6	— 3	0	+ 4	+ 8
Feb	8	10	11	11	10	8	6	4	2	0	2	6	8	10	11	11	10	8	6	4	2	0	2	6
Mar	+ 3	+ 4	+ 5	5	+ 4	+ 3	+ 3	+ 2	+ 1	0	— 1	— 3	— 3	— 4	— 5	— 5	— 4	— 3	— 3	— 2	— 1	0	+ 1	+ 3
April ...	— 2	— 2	— 2	— 2	— 2	— 2	— 2	— 1	— 0	0	+ 0	+ 1	+ 2	+ 2	+ 2	+ 2	+ 2	+ 2	+ 2	+ 1	+ 1	0	— 1	— 1
May	6	8	9	9	8	6	5	3	1	0	2	5	6	8	9	9	8	6	5	3	2	0	2	5
June	10	12	13	13	12	10	8	5	3	0	3	7	10	12	13	13	12	10	8	5	3	0	3	7
July	11	13	14	14	13	11	9	6	3	0	4	8	11	13	14	14	13	11	9	6	3	0	4	8
Aug	8	10	11	11	10	8	6	4	2	0	2	6	8	10	11	11	10	8	6	4	2	0	2	6
Sept	— 3	— 4	— 5	— 5	— 4	— 3	— 3	— 2	— 1	0	+ 1	+ 3	+ 3	+ 4	+ 5	+ 5	+ 4	+ 3	+ 3	+ 2	+ 1	0	— 1	— 3
Oct	+ 2	+ 2	+ 2	+ 2	+ 2	+ 2	+ 2	+ 1	+ 0	0	— 0	— 1	— 2	— 2	— 2	— 2	— 2	— 2	— 2	— 1	— 1	0	+ 1	+ 1
Nov	6	8	9	9	8	6	5	3	1	0	2	5	6	8	9	9	8	6	5	3	2	0	2	5
Dec	+10	+12	+13	+13	+12	+10	+ 8	+ 5	+ 3	0	— 3	— 7	—10	—12	—13	—13	—12	—10	— 8	— 5	— 3	0	+ 3	+ 7

NOTE.—For lower transits the signs must be reversed.

Example of the computation of a tidal ephemeris for the first part of January, 1871.

A	a	B	C	c	D	d	L′	e	f	g	h	i	j	B_0
d. h. m.	*m.*	*h. m.*	′ ″	″	°	″	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*
Dec. 30 6 55.0	+2.7	6 52.3	54 47	—1.0	+ 3	+11.6	44.4	13.0	11.3	0.4	14.3	0.1	2.8	9.0
31 7 36.5	3.2	7 33.3	54 22	0.7	8	10.9	53.6	12.1	13.3	1.5	14.6	0.6	2.2	9.0
Jan. 1 8 18.8	3.7	8 15.1	54 9	—0.3	12	9.9	58.3	10.9	14.4	2.5	14.8	1.2	1.7	9.0
2 9 2.7	4.2	8 58.5	54 5	0.0	16	8.5	58.4	10.0	15.0	4.0	14.7	2.0	1.3	9.0
3 9 48.9	4.7	9 44.2	54 10	+0.3	19	6.6	54.3	9.1	14.4	5.6	14.3	2.8	0.8	9.0
4 10 37.2	5.1	10 32.1	54 23	0.6	22	4.4	47.1	8.3	13.1	6.8	13.5	3.5	0.6	9.0
5 11 27.5	5.6	11 21.9	54 42	0.8	23	+1.7	37.4	7.6	11.4	7.7	12.7	4.0	0.5	9.1
6 12 19.2	6.0	12 13.9	55 6	1.0	23	—1.2	27.2	7.0	9.0	8.0	11.8	4.0	0.5	9.1

E	Δ^1	Δ^2	δ^1	k	l	m	t.h.w	Q	n	$\frac{1}{3}n$	A_2	Δ^1	Δ^2	δ^1	H_0+A_2	p	q	r	h.h.w
d. h. m.	*m*	*m.*	*m.*	*m.*	*m.*	*m.*	*m.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*				*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*
Jan. 1 7 30.3	52.9	. .	26.9	—0.8	+1.3	+0.5	31	3.35	—1.55	—.52	3.87	+.03	. .	—.00	23.96	+.18	+.22	+.40	24.32
19 57.2	26.4		26.0			+0.1	57				3.87	.01		+.03	23.95			.45	23.50
2 8 23.2	48.7	—4.2	24.8	1.2	0.9	—0.3	23	3.40	1.50	.50	3.90	.14	+.11	.06	23.99	.26	.24	.50	24.49
20 48.0	24.3	0.5	23.9			0.8	49				3.96	.07	.01	.08	24.05			.54	23.51
3 9 11.9	45.2	3.5	23.0	1.6	+0.4	1.2	11	3.61	1.29	.43	4.04	.19	.05	.08	24.13	.34	.24	.58	24.71
21 34.9	22.6	0.4	22.2			1.6	36				4.12	.09	.01	.11	24.21			.62	23.59
4 9.57.1	42.1	3.1	21.4	1.9	0.0	1.9	55	3.89	1.01	.34	4.23	.20	.01	.10	24.32	.41	.24	.65	24.97
22 18.5	21.0	0.4	20.7			2.3	21				4.33	.10	.00	.10	24.42			.68	23.74
5 10.39.2	39.9	2.2	20.3	2.2	—0.5	2.7	37	4.19	0.71	.24	4.43	.19	+.01	.09	24.52	.47	.24	.71	25.23
22 59.5	20.0	0.3	19.6			3.0	62				4.52	.09	.09	.10	24.61			.72	23.89
6 11 19.1	38.7	1.2	19.5	2.3	0.9	3.2	16	4.48	0.42	.14	4.62	.17	—.02	.08	24.71	.49	.23	.72	25.43
23 38.6	19.3	0.2	19.2			3.4	42				4.70	.08	.00	.09	24.79			.71	24.08
7 11 57.8	38.0	—0.7	19.1	2.3	1.2	3.5	54	4.74	—0.16	—.05	4.79	+.12	—.05	.07	24.88	.49	.20	.69	25.57
8 0 16.9	19.0	0.1	18.9			3.5	20				4.86	.06	.01	+.05	24.95			.66	24.29
12 35.8	. . .	. .	19.0	2.2	—1.3	3.5	32	4.92	+0.02	+.01	4.91	. .	. .		25.00	+.47	+.16	+.63	25.63

E′	k′	l′	m′	t.l.w	A'_2	$H_0-A'_2$	p′	q′	r′	h.l.w	M	S	Log. sin 2 B	Log. cos 2 B.	S cos 2 B	
															Log.	No.
d. h. m.	*m.*	*m.*	*m.*	*m.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*	*Ft.*				*Ft.*
Jan. 1 13 43.7	+0.8	+1.4	+2.2	46	3.86	16.23	+.11	+.08	+.19	16.42	4.15	0.92	9.644*n*	9.953*n*	9.917*n*	—0.83
2 2 10.2			2.5	8	3.88	16.21			.21	16.01						
14 35.6	1.2	1.5	2.7	38	3.93	16.16	.16	.06	.22	16.38	3.97	0.92	9.862	9.837	9.801	0.63
3 2 0.0			2.9	57	4.00	16.09			.23	15.86						
15 23.4	1.6	1.5	3.1	26	4.08	16.01	.21	+.03	.24	16.25	3.86	0.92	9.966	9.582	9.548	0.35
4 3 46.0			3.2	43	4.17	15.92			.25	15.67						
16 7.8	1.9	1.5	3.4	11	4.28	15.81	.25	.00	.25	16.06	3.79	0.92	10.000	8.090*n*	8.054*n*	—0.01
5 4 28.8			3.6	25	4.38	15.71			.25	15.46						
16 49.3	2.2	1.5	3.7	53	4.48	15.61	.28	—.03	.25	15.86	3.76	0.92	9.967	9.576*p*	9.540*p*	+0.35
6 5 9.3			3.7	6	4.57	15.52			.24	15.28						
17 28.8	2.3	1.4	3.7	32	4.66	15.43	.30	.06	.24	15.67	3.78	0.92	9.842	9.857	9.821	0.66
7 5 48.2			3.7	44	4.75	15.34			.23	15.11						
18 7.3	2.3	1.3	3.6	11	4.83	15.26	.30	.09	.21	15.47	3.86	0.93	9.501*n*	9.977	9.941	0.87
8 6 26.4			3.5	23	4.89	15.20			.19	15.01						
18 45.3	+2.2	+1.1	+3.3	49	4.93	15.16	+.28	—.11	+.17	15.33	4.00	0.93	9.083*p*	9.997*p*	9.965*p*	+0.92

E′—Continued.	M + S cos 2 B		Log. S sin 2 B	Log. tan β′	Log. M	Log. M²	M²	M² + S²	2 M S cos 2 B		M²+S²+2 M S cos 2 B		Log. Q
	No.	Log.							Log.	No.	No.	Log.	
d. h. m.	*Ft.*												
Jan. 1 13 43.7	3.32	0.521	9.608*p*	9.087*p*	0.618	1.236	17.22	18.07	0.836*n*	—6.85	11.22	1.050	0.525
2 2 10.2													
14 35.6	3.34	0.524	9.826	9.302	0.599	1.198	15.78	16.63	.701	5.02	11.61	1.065	0.532
3 2 0.0													
15 23.4	3.51	0.545	9.930	9.385	0.587	1.174	14.93	15.78	.436	2.73	13.05	1.116	0.558
4 3 46.0													
16 7.8	3.78	0.578	9.965	9.388	0.579	1.158	14.39	15.24	8.934*n*	—0.09	15.15	1.180	0.590
5 4 28.8													
16 49.3	4.11	0.614	9.933	9.319	0.575	1.150	14.13	14.99	9.416*p*	+2.61	17.60	1.245	0.622
6 5 9.3													
17 28.8	4.44	0.647	9.809	9.162	0.577	1.154	14.26	15.12	0.699	5.00	20.12	1.303	0.651
7 5 48.2													
18 7.3	4.73	0.675	9.468*p*	8.792*p*	0.587	1.174	14.93	15.79	0.829	6.74	22.53	1.352	0.676
8 6 26.4													
18 45.3	4.92	0.692	9.051*n*	8.359*n*	0.602	1.204	16.00	16.86	0.868*p*	+7.38	24.24	1.385	0.692